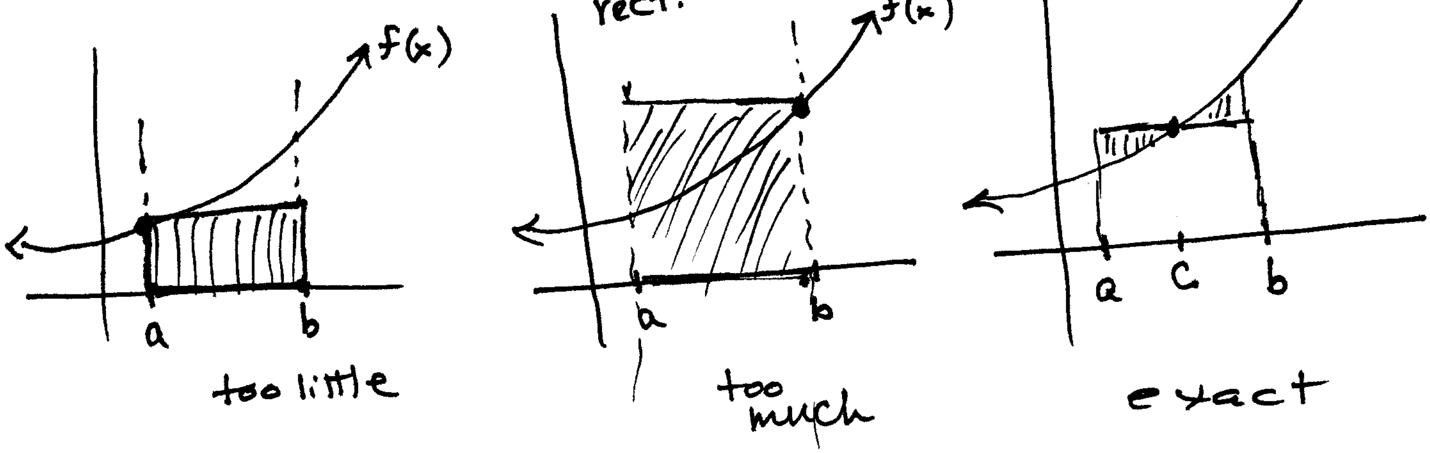
Mean Value Theorem for Integrals

If f is continuous on $[a,b]$, then there exists a # "c" in $[a,b]$ such that

$$\int_a^b f(x)\,dx = \underbrace{f(c)}_{\text{ht. of rect.}} \cdot \underbrace{(b-a)}_{\text{base length of rect.}}.$$



$$* \text{ average value } = f(c) = \frac{\int_a^b f(x)\,dx}{b-a} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex1 $\quad f(x) = 3x^2 - 2x \quad [1,4]$

Find the average value.

$$f(c) = \frac{1}{4-1}\int_1^4 (3x^2 - 2x)\,dx$$

$$= \frac{1}{3}\left[x^3 - x^2 + c \Big|_1^4 \right]$$

$$= \frac{1}{3}\left[64 - 16 + c - (1 - 1 + c) \right]$$

$$= \frac{1}{3}\left[48 \right] = 16$$

EX2 Use the MVT for Integrals to find $f(c)$ and c for $f(x) = 2x^2$ over $[0, 3]$.

$$f(c) = \frac{1}{3-0}\int_0^3 2x^2\,dx$$

$$= \frac{1}{3}\left[\frac{2}{3}x^3 + c\Big|_0^3\right]$$

$$= \frac{1}{3}\left[18 + c - (0 + c)\right]$$

$$\boxed{f(c) = 6}$$

$$6 = 2x^2$$

$$3 = x^2$$

$$x = \pm\sqrt{3}$$

$$\boxed{c = \sqrt{3}}$$

Find: $\frac{d}{dx}\int_4^x (2t+3)\,dt$

$$\frac{d}{dx}\left[t^2 + 3t + C\Big|_4^x\right]$$

$$\frac{d}{dx}\left[x^2 + 3x + C - (16 + 12 + C)\right]$$

$$\frac{d}{dx}\left[x^2 + 3x - 28\right]$$

$$2x + 3$$

$\frac{d}{dx}\int_5^x (2t+3)\,dt$

$$\frac{d}{dx}\left[t^2 + 3t + C\Big|_5^x\right]$$

$$\frac{d}{dx}\left[x^2 + 3x + C - (25 + 15 + C)\right]$$

$$\frac{d}{dx}\left[x^2 + 3x - 40\right]$$

$$2x + 3$$

$$\frac{d}{dx}\int_4^{x^5}(2t+3)\,dt$$

$$\frac{d}{dx}\left[\, t^2+3t+C\Big|_4^{x^5}\right]$$

$$\frac{d}{dx}\left[x^{10}+3x^5+C-(16+12+C)\right]$$

$$\frac{d}{dx}\left[x^{10}+3x^5-28\right]$$

$$10x^9+15x^4$$

$$5x^4(2x^5+3)$$

The Second Fundamental Theorem of Calculus

If f is cont. on an open interval I containing a, then for every x in I

$$\frac{d}{dx}\int_a^{x} f(t)\,dt = f(x)$$

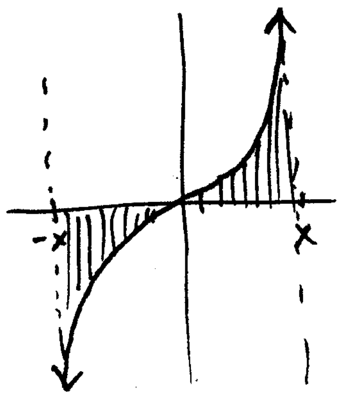
if this is a function of x, use the chain rule

EX 3 Find the derivative:

A. $F=\displaystyle\int_{\pi/2}^{x}\cos t\,dt$ $\qquad \displaystyle\frac{d}{dx}\int_{\pi/2}^{x}\cos t\,dt = \boxed{\cos x}$

B. $F = \int_{\pi/2}^{x^3} \cos t \, dt$ $\qquad \frac{d}{dx} \int_{\pi/2}^{x^3} \cos t \, dt = \boxed{3x^2 \cos x^3}$

C. $F = \int_{-x}^{x} t^3 dt$ $\qquad \frac{d}{dx} \int_{-x}^{x} t^3 dt$



$$\frac{d}{dx}\left[\int_{-x}^{0} t^3 dt + \int_0^x t^3 dt\right]$$

$$\frac{d}{dx}\left[-\int_0^{-x} t^3 dt + \int_0^x t^3 dt\right]$$

$$-(-x)^3 \cdot (-1) + x^3$$

$$-x^3 + x^3$$

$$\boxed{0}$$

EX4 Evaluate: $\int_{-3}^{x} \sqrt[5]{t^2}\, dt$

$\hookrightarrow t^{2/5}$

$$\frac{5}{7} t^{7/5} + c \Big|_{-3}^{x}$$

$$\frac{5}{7} x^{7/5} + c - \left(\frac{5}{7}(-3)^{7/5} + c\right)$$

$$\frac{5}{7} x^{7/5} - \frac{5}{7}(-3)^{7/5}$$