Alternating Series Test

- use with series that alternate signs

Let $a_n > 0$. Then $\sum (-1)^n a_n$ or $\sum (-1)^{n+1} a_n$ converges if:

① $\lim_{n\to\infty} a_n = 0$ **AND** ② $a_{n+1} \leq a_n$ for all n

EX 1 Ⓐ $\sum (-1)^{n+1} \cdot \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots$

① $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

② $\frac{1}{n+1} \leq \frac{1}{n}$ ✓ series converges by the Alt. Series Test

Ⓑ $\sum \frac{(-1)^n}{\ln(n+1)} = -\frac{1}{\ln 2} + \frac{1}{\ln 3} - \frac{1}{\ln 4} + \ldots$

① $\lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$ ✓

② $\frac{1}{\ln(n+2)} \leq \frac{1}{\ln(n+1)}$ ✓ series converges by the Alt. Series Test

Ⓒ $\sum \frac{(-1)^n \sqrt{n}}{\sqrt[3]{n}}$

① $\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt[3]{n}} = \lim_{n\to\infty} \frac{n^{1/2}}{n^{1/3}} = \lim_{n\to\infty} n^{1/6} = \infty$

alt. series test does not apply

n^{th} term test shows the series diverges

(D) $\sum \frac{1}{n} \cdot \underbrace{\sin\left(\frac{\pi}{2} n\right)}_{\text{sign change}} = 1 \cdot \sin\frac{\pi}{2} + \frac{1}{2}\cdot\sin\pi + \frac{1}{3}\cdot\sin\frac{3\pi}{2} + \frac{1}{4}\cdot\sin 2\pi + \frac{1}{5}\cdot\sin\frac{5\pi}{2}$

$= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$

(1) $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

(2) $\frac{1}{n+1} \le \frac{1}{n}$ ✓ converges by the alternating Series Test

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<u>Absolute Convergence</u> If $\sum |a_n|$ converges, then $\sum a_n$ converges.

<u>conditional Convergence</u> – occurs when $\sum |a_n|$ diverges but $\sum a_n$ converges

$\sum \frac{(-1)^n}{n}$ conv. by the A.S.T.

$\sum \left|\frac{(-1)^n}{n}\right| = \sum \frac{1}{n}$ p-series $p=1$ div. by the p-series test

$\boxed{\text{conditional convergence}}$

EX2 Determine absolute or conditional convergence.

(A) $\sum \frac{(-1)^n}{\sqrt{n}}$

① $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓

② $\frac{1}{\sqrt{n+1}} \le \frac{1}{\sqrt{n}}$ ✓

converges by the A.S.T.

$\sum \left| \frac{(-1)^n}{\sqrt{n}} \right| = \sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$

p-series $p = \frac{1}{2}$

$\frac{1}{2} < 1$

diverges by the p-series test

$\boxed{\text{conditional convergence}}$

(B) $\sum \frac{(-1)^n n}{n^3 - 1}$

① $\lim_{n\to\infty} \frac{n}{n^3-1} = 0$ ✓

② $\frac{n+1}{(n+1)^3 - 1} \le \frac{n}{n^3-1}$ ✓

converges by the A.S.T.

$\sum \left| \frac{(-1)^n n}{n^3-1} \right| = \sum \frac{n}{n^3-1}$

compare to $\sum \frac{n}{n^3} = \sum \frac{1}{n^2}$

p-series $p = 2$ $\quad 2 > 1$

converges by the p-series test

$$\lim_{n\to\infty} \frac{\frac{n}{n^3-1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^3}{n^3-1} = 1$$

pos. finite

$\sum \frac{n}{n^3-1}$ converges by the Limit Comparison Test

$\boxed{\text{absolute convergence}}$

1. n^{th} term test
2. special series?
   (p, geom., telesc.)
3. alt series test
4. comparison
5. integral