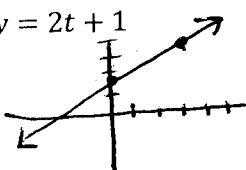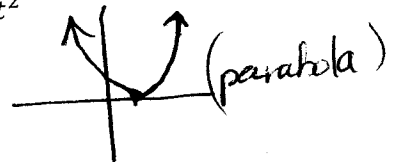## Practice 4.1: Arc Lengths & Parametrics Review

Part 1: Write the corresponding rectangular equation by eliminating the parameter, then sketch the curve represented by the parametric equations.

1) $x = 3t - 1, \quad y = 2t + 1$

$y = \frac{2}{3}x + \frac{5}{3}$
(line)

3) $x = t^3, \quad y = \frac{t^2}{2}$
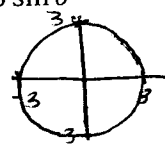
2) $x = t + 1, \quad y = t^2$

$y = (x-1)^2$ (parabola)

4) $x = \sqrt{t}, \quad y = t - 2$

5) $x = t - 1, \quad y = \frac{t}{t-1}$

6) $x = 2t, \quad y = |t - 2|$
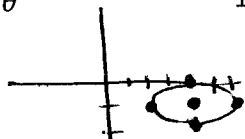
7) $x = e^t, \quad y = e^{3t} + 1$

8) $x = \sec\theta, \quad y = \cos\theta$

9) $x = 3\cos\theta, \quad y = 3\sin\theta$

$x^2 + y^2 = 9$
(circle)

10) $x = 4\sin 2\theta, \quad y = 2\cos 2\theta$

11) $x = 4 + 2\cos\theta, \quad y = -1 + \sin\theta$

$\frac{(x-4)^2}{4} + \frac{(y+1)^2}{1} = 1$
(ellipse)

13) $x = 4\sec\theta, \quad y = 3\tan\theta$

12) $x = 4 + 2\cos\theta, \quad y = -1 + 4\sin\theta$

14) $x = t^3, \quad y = 3\ln t$

15) $x = e^{-t}, \quad y = e^{3t}$

Part 2: Find the arc length of the graph of the function over the indicated interval.

16) $y = \frac{x^5}{10} + \frac{1}{6x^3}, \quad [1, 2]$

3.246

17) $y = \ln(\sin x), \quad \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

1.763

18) $y = \frac{1}{2}(e^x + e^{-x}), \quad [0, 2]$

3.627

1

Practice 4.2: Calculus on Parametrics

Find $\frac{dy}{dx}$.

1) $x = t^2, \quad y = 5 - 4t$ $\quad -\dfrac{2}{t}$

2) $x = \sin^2\theta, \quad y = \cos^2\theta$ $\quad -1$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, and find the slope and concavity (if possible) at the given value of the parameter.

Parametric Equations	Point	$\dfrac{dy}{dx}$	$\dfrac{d^2y}{dx^2}$	Slope	Concavity
3) $x = 2t, \quad y = 3t - 1$ (line)	$t = 3$	$\dfrac{3}{2}$	0	$\dfrac{3}{2}$	—
4) $x = t + 1, \quad y = t^2 + 3t$	$t = -1$	$2t+3$	2	1	concave up
5) $x = 2\cos\theta, \quad y = 2\sin\theta$	$\theta = \dfrac{\pi}{4}$	$-\cot\theta$	$\dfrac{-1}{2}\csc^3\theta$	-1	concave down
6) $x = 2 + \sec\theta, \quad y = 1 + 2\tan\theta$	$\theta = \dfrac{\pi}{6}$	$2\csc\theta$	$-2\cot^3\theta$	4	concave down ⌣
7) $x = \cos^3\theta, \quad y = \sin^3\theta$	$\theta = \dfrac{\pi}{4}$	$-\tan\theta$	$\dfrac{1}{3}\sec^4\theta\csc\theta$	-1	concave up

8) Find all points (if any) of horizontal and vertical tangency to the curve $x = t^3 - 3t$, $y = 3t^2 - 9$.

horiz: $(-1, 0)$

vert: none

Find the arc length of the curve on the given interval.

Parametric Equations	Interval
9) $x = t^2, \quad y = 2t$	$0 \le t \le 2$

5.916

| 10) $x = e^{-t}\cos t, \quad y = e^{-t}\sin t$ | $0 \le t \le \dfrac{\pi}{2}$ |

2

1.120