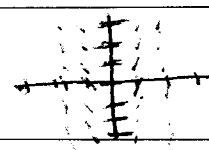
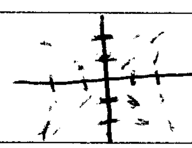
Differential Equations Review

OPPS Name:______________________

1. If $\dfrac{dy}{dx} = \dfrac{7x^2}{y^3}$ and $y(3) = 2$, find an equation for y in terms of x.	$y = \sqrt[4]{\frac{28}{3}x^3 - 236}$
2. If $\dfrac{dy}{dx} = 5x^2 y$ and $y(0) = 6$, find an equation for y in terms of x.	
3. If $\dfrac{dy}{dx} = \dfrac{1}{y + x^2 y}$ and $y(0) = 2$, find an equation for y in terms of x.	$y = \sqrt{2\arctan x + 4}$
4. If $\dfrac{dy}{dx} = \dfrac{e^x}{y^2}$ and $y(0) = 1$, find an equation for y in terms of x.	
5. If $\dfrac{dy}{dx} = \dfrac{y^2}{x^3}$ and $y(1) = 2$, find an equation for y in terms of x.	$y = 2x^2$
6. If $\dfrac{dy}{dx} = \dfrac{\sin x}{\cos y}$ and $y(0) = \dfrac{3\pi}{2}$, find an equation for y in terms of x.	
7. The rate of growth of the volume of a sphere is proportional to its volume. If the volume of the sphere is initially 36π ft^3, and expands to 90π ft^3 after 1 second, find the volume of the sphere after 3 seconds.	$\frac{1125\pi}{2}$ ft^3
8. Use Euler's Method, with $h = 0.25$, to estimate $y(1)$ if $y' = y - x$ and $y(0) = 2$.	
9. Use Euler's Method, with $h = 0.2$, to estimate $y(1)$ if $y' = -y$ and $y(0) = 1$.	.32768
10. Use Euler's Method, with $h = 0.1$, to estimate $y(0.5)$ if $y' = 4x^3$ and $y(0) = 0$.	
11. Sketch the slope field for $\dfrac{dy}{dx} = 2x$.	
12. Sketch the slope field for $\dfrac{dy}{dx} = -\dfrac{x}{y}$.	
13. Sketch the slope field for $\dfrac{dy}{dx} = \dfrac{x}{y}$.	

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Logistic Growth Problems
AP Calculus BC

Name: ____________________
Date: ____________________

1

The carrying capacity for deer in a particular small town is 2,200, and the rate of increase in their numbers is proportional to both the number, n, of deer and $2{,}200 - n$. If there were 1,000 deer one month ago and 1,150 deer now, how many months will it take the deer to number 2,100?

2

Guppy Population A 2000-gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank. Assume that the rate of growth of the population is

$$\frac{dP}{dt} = 0.0015P(150 - P),$$

where time t is in weeks.

(a) Find a formula for the guppy population in terms of t.

(b) How long will it take for the guppy population to be 100? 125?

3

Gorilla Population A certain wild animal preserve can support no more than 250 lowland gorillas. Twenty-eight gorillas were known to be in the preserve in 1970. Assume that the rate of growth of the population is

$$\frac{dP}{dt} = 0.0004P(250 - P),$$

where time t is in years.

(a) Find a formula for the gorilla population in terms of t.

(b) How long will it take for the gorilla population to reach the carrying capacity of the preserve?

a) $p(t) = \dfrac{250}{1 + 7.9825e^{-.1t}}$

b) won't quite reach 250 (horiz. asymptote)

4

Suppose that the growth of a population $y = y(t)$ is given by the logistic equation

$$y = \frac{60}{5 + 7e^{-t}} = \frac{60}{5(1+\frac{7}{5}e^{-t})} = \frac{12}{1+\frac{7}{5}e^{-t}}$$

(a) What is the population at time $t = 0$?
(b) What is the carrying capacity L?
(c) What is the constant k?
(d) When does the population reach half of the carrying capacity?
(e) Find an initial-value problem whose solution is $y(t)$.

a) 5
b) 12
c) 1
d) 0.336
e) $\frac{dy}{dt} = 1y\left(1 - \frac{y}{12}\right)$

5

Suppose that the growth of a population $y = y(t)$ is given by the logistic equation

$$y = \frac{1000}{1 + 999e^{-0.9t}}$$

(a) What is the population at time $t = 0$?
(b) What is the carrying capacity L?
(c) What is the constant k?
(d) When does the population reach 75% of the carrying capacity?
(e) Find an initial-value problem whose solution is $y(t)$.

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