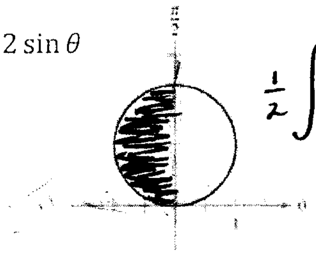
Practice 4.7: Polar, Area, & Arc Length

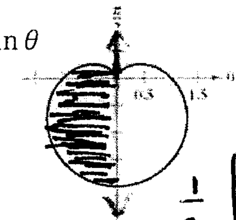
Write an integral that represents the area of the shaded region in the figure. Do not evaluate the integral.

1) $r = 2\sin\theta$



$\frac{1}{2}\int_{\pi/2}^{\pi} (2\sin\theta)^2\, d\theta$

2) $r = 1 - \sin\theta$



$\frac{1}{2}\int_{\pi/2}^{3\pi/2} (1-\sin\theta)^2\, d\theta$

3) Find the area of the region bounded by the graph of the polar equation $r = 8\sin\theta$ using the following;

a) a geometric formula $A = \pi r^2 = \pi(4)^2$ 16π

b) integration $\frac{1}{2}\int_0^{\pi} (8\sin\theta)^2\, d\theta = \cancel{16\pi}$ 50.265

Find the area of the region.

4) One petal of $r = 2\cos 3\theta$. 1.047

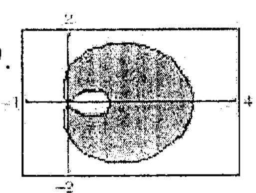
5) One petal of $r = \cos 2\theta$. .393

6) Interior of $r = 1 - \sin\theta$. 4.712

7) Inner loop of $r = 1 + 2\cos\theta$. .544

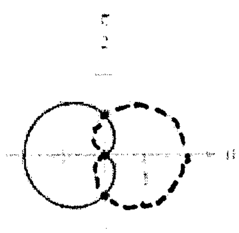
8) Between the loops of $r = 1 + 2\cos\theta$.

8.338



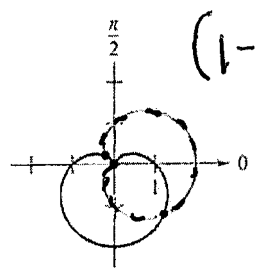
Find the points of intersection of the graphs of the equations.

9) $r = 1 + \cos\theta \quad r = 1 - \cos\theta$



$(1, \pi/2)$
$(1, 3\pi/2)$
$(0, 0)$

10) $r = 1 + \cos\theta \quad r = 1 - \sin\theta$



$\left(1 - \frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$
$\left(1 + \frac{\sqrt{2}}{2}, \frac{7\pi}{4}\right)$
$(0, 0)$

11) $r = 4 - 5\sin\theta$, $r = 3\sin\theta$

$(3/2, \pi/6)$
$(3/2, 5\pi/6)$
$(0, 0)$

12) $r = \frac{\theta}{2}$, $r = 2$

$(2, 4)$
$(-2, -4)$

13) $r = 4\sin 2\theta$, $r = 2$

$(2, \pi/12)$ $(2, 5\pi/12)$
$(2, 7\pi/12)$ $(2, 11\pi/12)$
$(2, 13\pi/12)$ $(2, 17\pi/12)$
$(2, 19\pi/12)$ $(2, 23\pi/12)$

14) Find the length of the curve $r = 1 + \sin\theta$ over $[0, 2\pi]$.

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