AP Calculus BC **Practice 3.6: Error**

Error in Alternating Series

1. How many terms of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^4}$ are needed to approximate the sum of the series with error < 0.001?
   a. 3
   b. 4
   c. 5
   d. 6
   e. 7

2. Consider the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^p}$
   a. Determine the values for which the series converges. $p > 0$
   b. Find the maximum value of the error if k terms are used to approximate the sum of the series. $\frac{1}{(k+1)^p}$

3. Given the following series: $\sum_{n=1}^{10}(-1)^{n+1}\frac{1}{n^n}$
   a. Find the sum. .78343
   b. What is the maximum value of the error if the sum $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^n}$ is approximated by the answer to part (a)? 3.505×10^{-12}
   c. If the sum of the first twenty terms was used instead in part (a), would the error in part (b) be increased or decreased? Explain your answer. more terms ⇒ better approx. ⇒ less error

4. Given the alternating series $\sum_{n=0}^{\infty}(-0.25)^n$
   a. Find the error when the series is approximated by the first four terms of the series. $\frac{1}{256}$
   b. Find the exact sum of the series and use it to find the actual error between exact sum and the approximation using the first four terms of the series. Compare your answer to the answer in part (a). 4/5 .003125

5. Find an upper bound for the error when the sum is estimated using the first ten terms of the following series: $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$ with the first ten terms of the series. $\frac{1}{11}$

6. What is the least positive integer n that will make the error estimate of $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^3}$ less than 0.001? 11

7. Find the error for estimating sin (0.5) by the third-degree Taylor polynomial for sin x.

   .00002604

8. The error in estimating e^{-2} using five terms of the Taylor series for e^{-x} is not greater than __________?

   $\frac{2^5}{5!}$

12

Lagrange Error Bound

9. Using the formula for the error E, what is the maximum value of the error in approximating ln 1.2 with a Taylor polynomial of degree 3 centered at x =1?
   a. 0.000345
   b. 0.0004
   c. 0.00666
   d. 0.1813
   e. 0.1827

10. Find the remainder when $\frac{1}{\sqrt{e}}$ is approximated by a third-degree MacLaurin polynomial.
   a. 0.0024
   b. 0.0208
   c. 0.2916
   d. 0.5833
   e. 1.6667