AP Calculus BC   **Practice 3.8: Test Review**

Work the following on notebook paper. Use your calculator only when necessary.

1. Find the radius and interval of convergence: $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$ R.O.C = ∞
I.O.C $(-\infty, \infty)$

2. (a) Find the interval of convergence: $f(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)^2}$. $[-1, 1]$

(b) Write the first four nonzero terms and the general term for $f'(x)$, and find its interval of convergence.

$1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \ldots + \frac{x^n}{n+1} + \ldots$ $[-1, 1)$

3. (a) Find a power series for $f(x) = \frac{1}{1+x^2}$ centered at $x = 0$. Write the first four nonzero terms and the general term.

b. $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots + \frac{(-1)^n x^{2n+1}}{2n+1} + \ldots$

(b) Use your answer to (a) to find the first four nonzero terms and the general term for $g(x) = \arctan x$.

a. $1 - x^2 + x^4 - x^6 + \ldots + (-1)^n x^{2n} + \ldots$

(c) Use your answer to (b) to approximate $\arctan\frac{1}{3}$, using $R_N \le 0.001$. Justify your answer.

.321

For problems 4 – 5, write the first four nonzero terms and the general term.

4. Maclaurin series for $f(x) = \sin(x^3)$ $x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \ldots \frac{(-1)^n (x^3)^{2n+1}}{(2n+1)!} + \ldots$

5. Power series for $g(x) = \frac{x}{1+2x}$ centered at $x = 0$

$x - 2x^2 + 4x^3 - 8x^4 + \ldots + (-1)^n 2^n x^{n+1} + \ldots$

6. Suppose $f(x)$ is approximated near $x = 0$ by a fifth-degree Taylor polynomial $P_5(x) = 2x - 5x^3 + 4x^5$. Give the value of:

(a) $f''(0)$ 0

(b) $f'''(0)$ -30

(c) $f^{(5)}(0)$ 480

7. Use power series to evaluate $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$.   2

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8. Use a Taylor series of degree 5 for $\sin x$ about $x = 0$ to estimate $\int_0^1 \frac{\sin x}{x}\,dx$ with an error less than 0.001. Justify your answer. $.946$

9. The function f has derivatives of all orders for all real numbers x. Assume $f(3) = -5,\ f'(3) = 2,\ f''(3) = -7,\ f'''(3) = 9.$

(a) Write the third-degree Taylor polynomial for f about $x = 3$, and use it to approximate $f(2.6)$.

$-5 + 2(x-3) - \frac{7}{2!}(x-3)^2 + \frac{9}{3!}(x-3)^3$ $\quad -6.456$

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 5$ for all x in the closed interval [2.6, 3]. Use the Lagrange error bound on the approximation to $f(2.6)$ found in part (a) to explain whether or not $f(2.6)$ can equal -6. $\quad$ e.b. = .005333 $\quad -6.456 \pm$ eb.

(c) Write the fourth-degree Taylor polynomial, $Q(x)$, for $g(x) = f(x^2 + 3)$ about $x = 0$. $\quad -5 + 2x^2 - \frac{7}{2}x^4$

(d) Use your answer to (c) to determine whether g has a relative maximum, a relative minimum, or neither at $x = 0$. Justify your answer.

$g'(x) = 0$

$x = 0, \pm\sqrt{2/7}$

g' − to +

g decr to incr

10. The Taylor series about $x = 4$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 4$ is given by $f^{(n)}(4) = \frac{(-1)^n n!}{3^n(n+1)}$ for $n \ge 1$ and $f(4) = 2$.

(a) Write the third-degree Taylor polynomial for f about $x = 4$. $\quad 2 - \frac{1}{6}(x-4) + \frac{1}{27}(x-4)^2 - \frac{1}{108}(x-4)^3$

(b) Find the radius of convergence. $\quad 3$

(c) Use the series found in (a) to approximate $f(5)$ with an error less than 0.02. $\quad 1.870$

11. Solve the equation $y' = x + 2xy$ when y(0) = 1. $\quad y = \frac{3e^{x^2} - 1}{2}$

12. Evaluate: $\frac{d}{dt}\left(\int_0^{2t} \frac{1-\cos x}{x}\,dx\right)$ $\quad \frac{1 - \cos 2t}{t}$

13. Find the slope of the tangent to the graph of $\ln y + e^x = y$. $\quad \frac{dy}{dx} = \frac{ye^x}{y-1}$

14. Find the 4^{th}-degree Taylor polynomial for $y = e^{\frac{1}{4}x}$, centered at x = 0.

$1 + \frac{1}{4}x + \frac{1}{32}x^2 + \frac{1}{384}x^3 + \frac{1}{6144}x^4$

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15. Evaluate: $\int_1^{\infty} \frac{\ln x}{x}\,dx$ div

16. Determine if the following series converge or diverge:

A. $\sum_{n=1}^{\infty} n^2$ div

B. $\sum_{n=1}^{\infty} \frac{2n^2 - 1}{n^2 + 1}$ div

C. $\sum_{n=1}^{\infty} \frac{kn^2}{n!}$, where k is a constant conv

D. $\sum_{n=1}^{\infty} \frac{1}{n^2 - 3}$ conv

E. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln n}$ conv