## 

Work the following on notebook paper. Use your calculator only when necessary.

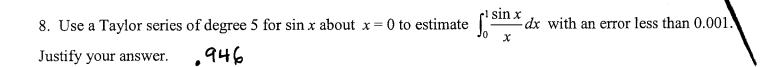
- $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!} \qquad \text{R.O.C} = \infty$   $\text{T.o.c.} (-\infty, \infty)$ 1. Find the radius and interval of convergence:
- 2. (a) Find the interval of convergence:  $f(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)^2}$ .
  - (b) Write the first four nonzero terms and the general term for f'(x), and find its interval of convergence.

$$1 + \frac{\times}{2} + \frac{\times^{3}}{3} + \frac{\times^{3}}{4} + \dots + \frac{\times^{n}}{n+1} + \dots \qquad [-1,1]$$

- 3. (a) Find a power series for  $f(x) = \frac{1}{1+x^2}$  centered at x = 0. Write the first four nonzero terms and the general term.
  - neral term.  $x \frac{x^3}{31} + \frac{x^5}{5} \frac{x^7}{7} + \dots + \frac{(-1)^n x^{2n+1}}{2n+1} + \dots$ (b) Use your answer to (a) to find the first four nonzero terms and the general term for  $g(x) = \arctan x$ .

    (c) Use your answer to (b) to approximate  $\arctan \frac{1}{3}$ , using  $R_N \le 0.001$ . Justify your answer.

- rightharpoons For problems 4 5, write the first four nonzero terms and the general term.
  - 4. Maclaurin series for  $f(x) = \sin(x^3)$   $\times 3 \frac{\times 9}{3!} + \frac{\times 15}{5!} \frac{\times^{21}}{7!} + \dots + \frac{(-1)^n (\chi^3)^{2n+1}}{(2n+1)!} + \dots$
  - 5. Power series for  $g(x) = \frac{x}{1+2x}$  centered at x = 0 $x - 2x^{2} + 4x^{3} - 8x^{4} + ... + (-1)^{h} 2^{n} x^{n+1} + ...$
  - 6. Suppose f(x) is approximated near x = 0 by a fifth-degree Taylor polynomial  $P_5(x) = 2x 5x^3 + 4x^5$ . Give the value of:
    - (a) f''(0)480
  - 7. Use power series to evaluate  $\lim_{x\to 0} \frac{e^x e^{-x}}{x}$ .



- 9. The function f has derivatives of all orders for all real numbers x. Assume f(3) = -5, f'(3) = 2, f''(3) = -7, f'''(3) = 9.
- (a) Write the third-degree Taylor polynomial for f about x = 3, and use it to approximate f(2.6).
- $-5+2(x-3)-\frac{7}{21}(x-3)^2+\frac{9}{31}(x-3)^3$  -6.456 (b) The fourth derivative of f satisfies the inequality  $|f^{(4)}(x)| \le 5$  for all x in the closed interval [2.6, 3]. Use the Lagrange error bound on the approximation to f(2.6) found in part (a) to explain whether or not f(2.6)e.b. = .005333 can equal -6. -6.456 ± eb.
- (c) Write the fourth-degree Taylor polynomial, Q(x), for  $g(x) = f(x^2 + 3)$  about x = 0.

  (d) Use your answer to (c) to determine whether g has a relative maximum, a relative minimum, or neither at
- g'(x) = 0  $x = 0, \pm \sqrt{2/7}$  g' = 0 g' = 0 g' = 0x = 0. Justify your answer.
- 10. The Taylor series about x = 4 for a certain function f converges to f(x) for all x in the interval of convergence. The nth derivative of f at x = 4 is given by  $f^{(n)}(4) = \frac{(-1)^n n!}{3^n (n+1)}$  for  $n \ge 1$  and f(4) = 2.
- (a) Write the third-degree Taylor polynomial for f about x = 4.  $2 \frac{1}{6} \left( x 4 \right) + \frac{1}{27} \left( x 4 \right)^2 \frac{1}{108} \left( x 4 \right)$
- (b) Find the radius of convergence.
- (c) Use the series found in (a) to approximate f(5) with an error less than 0.02.
- 11. Solve the equation y' = x + 2xy when y(0) = 1.
- 12. Evaluate:  $\frac{d}{dt} \left( \int_{1}^{2t} \frac{1 \cos x}{x} dx \right)$
- 13. Find the slope of the tangent to the graph of  $\ln y + e^x = y$ .
- 14. Find the 4<sup>th</sup>-degree Taylor polynomial for  $y = e^{\frac{1}{4}x}$ , centered at x = 0. 16

- 15. Evaluate:  $\int_{1}^{\infty} \frac{\ln x}{x} dx \qquad \text{div}$
- 16. Determine if the following series converge or diverge:
  - A.  $\sum_{n=1}^{\infty} n^2$  div
  - B.  $\sum_{n=1}^{\infty} \frac{2n^2-1}{n^2+1}$  d: V
  - C.  $\sum_{n=1}^{\infty} \frac{kn^2}{n!}$ , where k is a constant **ConV**
  - D.  $\sum_{n=1}^{\infty} \frac{1}{n^2 3}$
  - E.  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln n}$  Conv