Antiderivatives

Given a derivative, find the original function.

derivative $f'(x)$	original function $f(x)$
$y' = 2x$	$y = x^2 + C$
$y' = x$	$y = \frac{1}{2}x^2 + C$
$y' = x^2$	$y = \frac{1}{3}x^3 + C$
$y' = 5x^4$	$y = x^5 + C$
$y' = \frac{1}{x^2} = x^{-2}$	$y = -x^{-1} + C = -\frac{1}{x} + C$
$y' = \frac{1}{x^3} = x^{-3}$	$y = \frac{-1}{2}x^{-2} + C = \frac{-1}{2x^2} + C$
$y' = \cos x$	$y = \sin x + C$

differential equation — involves x, y & derivatives of y

$$\frac{dy}{dx} = f'(x)$$

$$\int 1\,dy = \int f'(x)\,dx$$

variable of integration

$$y = f(x) + C$$

indefinite integration

EX1 Find the general solution: $y' = 3x^2$

$$\frac{dy}{dx} = 3x^2$$

$$\int 1\,dy = \int 3x^2\,dx$$

$$\boxed{y = x^3 + C}$$

EX2 Evaluate:

A. $\int x^4\,dx = \frac{1}{5}x^5 + C$

B. $\int (x^2 - 2x + 3)\,dx = \frac{1}{3}x^3 - x^2 + 3x + C$

C. $\int (\sqrt[4]{x^3} + 1)\,dx = \int (x^{3/4} + 1)\,dx = \frac{4}{7}x^{7/4} + x + C$

D. $\int \frac{x^2+1}{x^2}\,dx = \int (1 + x^{-2})\,dx = x - 1x^{-1} + C$

$= x - \frac{1}{x} + C$

E. $\int (2t^2 - 1)^2\,dt = \int (4t^4 - 4t^2 + 1)\,dt = \frac{4}{5}t^5 - \frac{4}{3}t^3 + t + C$

F. $\int (t^2 - \sin t)\,dt = \frac{1}{3}t^3 + \cos t + C$

G. $\int (\theta^2 + \sec^2\theta)\,d\theta = \frac{1}{3}\theta^3 + \tan\theta + C$

H. $\int \sec y(\tan y - \sec y)\,dy = \int (\sec y \tan y - \sec^2 y)\,dy$

$= \sec y - \tan y + C$

I. $\displaystyle\int \frac{\sin x}{1-\sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx$

$\displaystyle = \int \tan x \cdot \sec x \,dx$

$= \sec x + C$

J. $\displaystyle\int \frac{\sqrt{x}+3}{x^2}\,dx = \int \left(x^{-\frac{3}{2}} + 3x^{-2}\right)dx$

$= -2x^{-\frac{1}{2}} - 3x^{-1} + C$

$\displaystyle = \frac{-2}{\sqrt{x}} - \frac{3}{x} + C$

EX 3 If $f'(s) = 6s - 8s^3$ and $f(2) = 3$ find $f(s)$.

$\displaystyle \frac{dy}{ds} = 6s - 8s^3$ $\qquad (2,3)$ $\quad x \; y$ $\qquad$ initial condition

$\int 1\,dy = \int (6s - 8s^3)\,ds$

$y = 3s^2 - 2s^4 + C$ $\leftarrow$ general soln.

$3 = 3(2)^2 - 2(2)^4 + C$

$3 = 12 - 32 + C$

$23 = C$

$\boxed{f(s) = 3s^2 - 2s^4 + 23}$ $\leftarrow$ particular soln.