Arc Length

Let $y = f(x)$ represent a smooth curve on the interval $[a, b]$.

arc length $= s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

or

$$\int_a^b \sqrt{\left[\frac{dx}{dx}\right]^2 + \left[\frac{dy}{dx}\right]^2}\, dx$$

EX 1 Find the arc length.

(A) $f(x) = \frac{4\sqrt{2}}{3} x^{3/2} - 1, \quad 0 \le x \le 1$

$f'(x) = \frac{4\sqrt{2}}{3} \cdot \frac{3}{2} x^{1/2} = 2\sqrt{2}\, x^{1/2} = 2\sqrt{2x}$

a.l. $= \int_0^1 \sqrt{1 + (2\sqrt{2x})^2}\, dx = \int_0^1 \sqrt{1 + 8x}\, dx$

$= 2.167$

(B) $f(x) = \frac{x^3}{6} + \frac{1}{2x} \rightarrow (2x)^{-1}, \quad [\frac{1}{2}, 2]$

$f'(x) = \frac{1}{2}x^2 - 1(2x)^{-2} \cdot 2 = \frac{1}{2}x^2 - \frac{1}{2x^2}$

a.l. $= \int_{1/2}^2 \sqrt{1 + \left(\frac{1}{2}x^2 - \frac{1}{2x^2}\right)^2}\, dx = 2.0625 = \frac{33}{16}$

(C) $(y-1)^3 = x^2, \quad [0, 8]$

$y - 1 = x^{2/3}$

$y = 1 + x^{2/3}$

$y' = \frac{2}{3} x^{-1/3}$

a.l. $= \int_0^8 \sqrt{1 + \left(\frac{2}{3}x^{-1/3}\right)^2}\, dx$

9.073

(D) $y = \ln(\cos x)$, $[0, \pi/4]$

$$y' = \frac{1}{\cos x} \cdot -\sin x = -\tan x$$

$$\text{a.l.} = \int_0^{\pi/4} \sqrt{1 + (-\tan x)^2}\, dx = .881$$

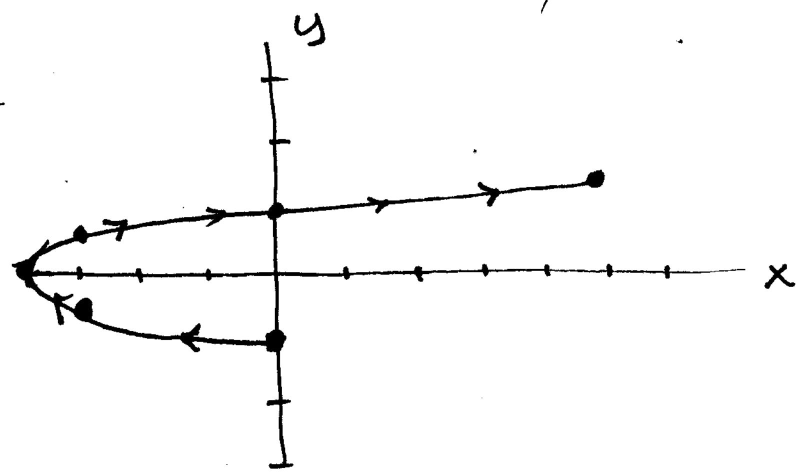
<u>Parametric Equations</u> – when x & y are given as a function of t

↑ "parameter"

$$x = t^2 - 4 \qquad -2 \le t \le 3$$
$$y = \frac{t}{2}$$

graphing parametric eqns gives a curve which has a direction/orientation

t	x	y
-2	0	-1
-1	-3	$-1/2$
0	-4	0
1	-3	$1/2$
2	0	1
3	5	$3/2$



when you "eliminate the parameter", you find a <u>familiar</u> <u>rectangular</u> <u>eqn</u> that represents the graph of the parametric eqns.

EX 2 Eliminate the parameter.

1. solve for t in one eqn
2. substitute that expression into the other eqn.

(A) $x = t^2 - 4$

$y = \frac{t}{2}$

$\to t = 2y$

$x = (2y)^2 - 4$

$\boxed{x = 4y^2 - 4}$

(B) $x = 3\cos\theta$

$y = 4\sin\theta$

$\frac{x}{3} = \cos\theta \qquad \frac{y}{4} = \sin\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{y}{4}\right)^2 + \left(\frac{x}{3}\right)^2 = 1$

$\boxed{\frac{x^2}{9} + \frac{y^2}{16} = 1}$

ellipse

center (0,0)