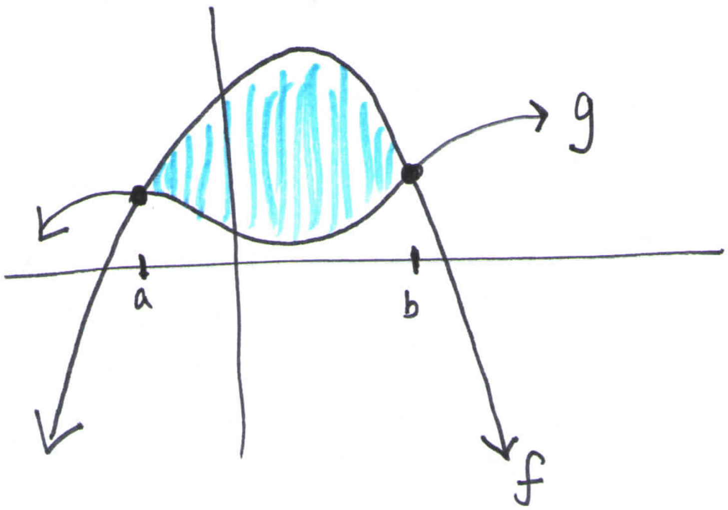
Area Between 2 Curves



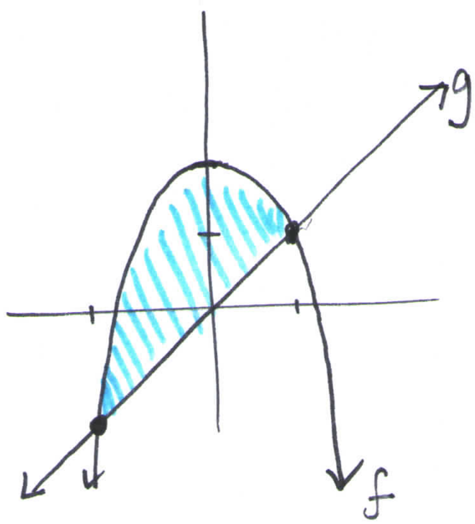
area of shaded region

$$\int_a^b \left[f(x) - g(x) \right] dx$$

where $f(x) > g(x)$ for $[a, b]$

Find the area between the curves:

1. $f(x) = 2 - x^2$ and $g(x) = x$



$$2 - x^2 = x$$
$$0 = x^2 + x - 2$$
$$0 = (x+2)(x-1)$$
$$x = -2, \quad x = 1$$

$$\int_{-2}^{1} (2 - x^2 - x)\, dx$$

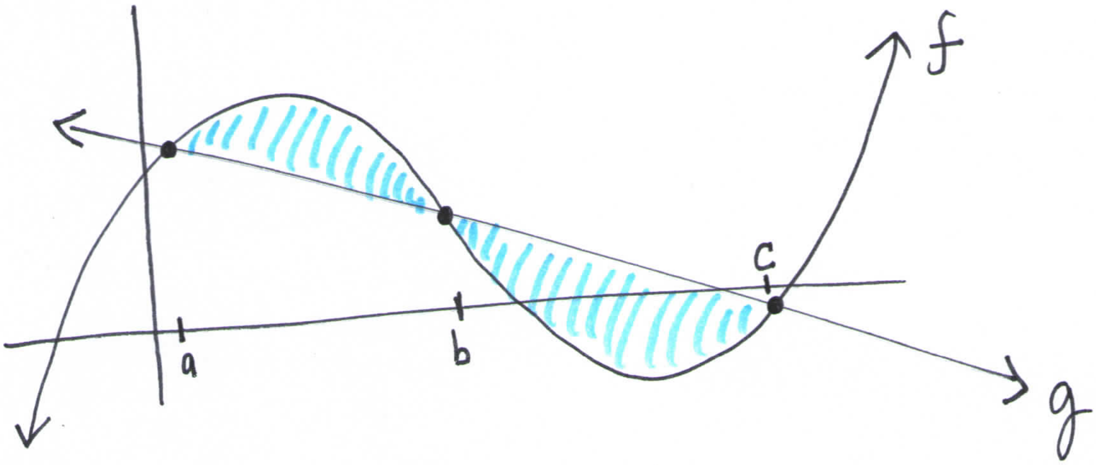
$$2x - \frac{1}{3}x^3 - \frac{1}{2}x^2 + C \Big|_{-2}^{1}$$

$$2 - \frac{1}{3} - \frac{1}{2} - \left(-4 + \frac{8}{3} - 2\right)$$

$$2 - \frac{1}{3} - \frac{1}{2} + 4 - \frac{8}{3} + 2$$

$$\boxed{\frac{9}{2}}$$

Sometimes curves intersect in more than 2 points...



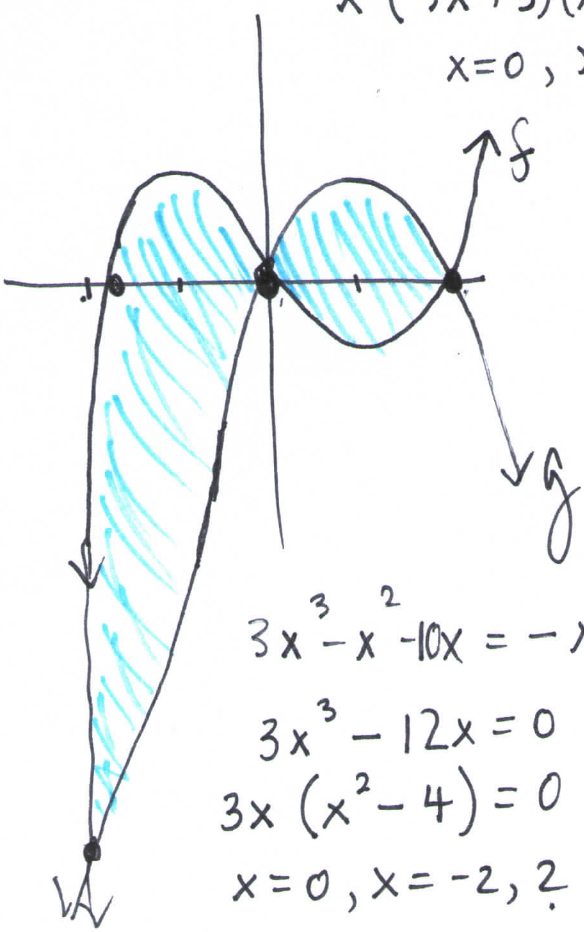
$$\int_a^b [f(x)-g(x)]\,dx \quad + \quad \int_b^c [g(x)-f(x)]\,dx$$

(2) $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$

$x(3x^2 - x - 10) = 0$ $\qquad$ $-x(x-2) = 0$

$x(3x+5)(x-2) = 0$ $\qquad$ $x = 0,\ x = 2$

$x = 0,\ x = -5/3,\ x = 2$



$3x^3 - x^2 - 10x = -x^2 + 2x$

$3x^3 - 12x = 0$

$3x(x^2 - 4) = 0$

$x = 0,\ x = -2,\ 2$

$$\int_{-2}^{0} [(3x^3 - x^2 - 10x) - (-x^2 + 2x)]\,dx \ +$$

$$\int_0^2 [(-x^2 + 2x) - (3x^3 - x^2 - 10x)]\,dx$$

$$= \int_{-2}^{0} (3x^3 - 12x)\,dx + \int_0^2 (-3x^3 + 12x)\,dx$$

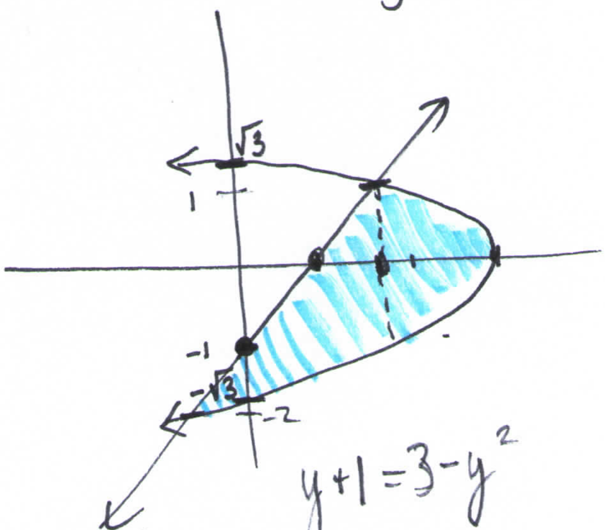
$$= \frac{3}{4}x^4 - 6x^2 + C \Big|_{-2}^{0} + \frac{-3}{4}x^4 + 6x^2 + C \Big|_0^2$$

$$= 0 - (12 - 24) \quad + \quad -12 + 24 - (0)$$

$$= 12 + 12 = \boxed{24}$$

③ $f(y) = y+1$ and $g(y) = 3-y^2$

$x = y+1$ $\quad y = x-1$ $\qquad x = 3-y^2$



right − left

$$\int_{-2}^{1}\Big[\underbrace{(3-y^2)-(y+1)}_{-y^2-y+2}\Big]\,dy$$

$y+1 = 3-y^2$

$y^2+y-2=0$

$(y+2)(y-1)=0$

$y=-2,\ y=1$

$$-\frac{1}{3}y^3-\frac{1}{2}y^2+2y+C\Big|_{-2}^{1}$$

$$-\frac{1}{3}-\frac{1}{2}+2-\left(\frac{8}{3}-2-4\right)$$

$$-\frac{1}{3}-\frac{1}{2}+2-\frac{8}{3}+2+4$$

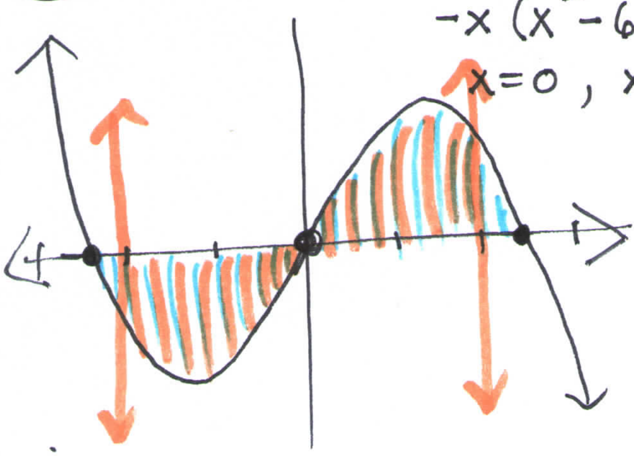
$$\boxed{\frac{9}{2}}$$

④ $f(x) = -x^3+6x$ and x-axis on $[-2,2]$

$y=0$

$-x(x^2-6)=0$

$x=0,\ x=\pm\sqrt{6}$



$$2\int_0^2\big[(-x^3+6x)-0\big]\,dx$$

$$2\left[-\frac{1}{4}x^4+3x^2+C\Big|_0^2\right]$$

$$2\big[-4+12-(0)\big]$$

$$\boxed{16}$$