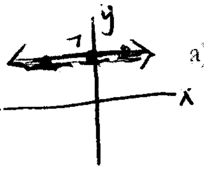
Notes--Basic Derivative Rules

Taking derivatives is a a process that is vital in calculus. In order to take derivatives, there are rules that will m the process simpler than having to use the definition of the derivative.

1. The constant rule: The derivative of a constant function is 0. That is, if c is a real number, then $\frac{d}{dx}[c]=0$.



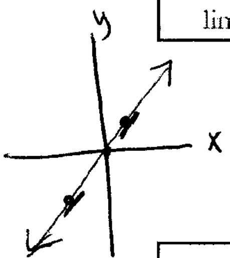
a) $y=7$ — $y'=0$

b) $f(x)=0$ — $f'(x)=0$

c) $s(t)=-8$ — $s'(t)=0$

d) $y=a\pi^3$ — $\frac{dy}{dx}=0$

2. The single variable rule: The derivative of x is 1. $\frac{d}{dx}[x]=1$. This is consistent with the fact that the slope of the line $y=x$ is 1.



a) $y=x$ — $y'=1$

b) $f(x)=x$ — $f'(x)=1$

c) $s(t)=t$ — $s'(t)=1$

3. The power rule: If n is a rational number then the function x^n is differentiable and $\frac{d}{dx}[x^n]=nx^{n-1}$.

Take the derivatives of the following. Use correct notation.

$y=x^1$

$y'=1\cdot x^0$

$1\cdot 1$

1

a) $y=x^2$

$y'=2x^1$

b) $f(x)=x^6$

$f'(x)=6x^5$

c) $s(t)=t^{30}$

$s'(t)=30t^{29}$

d) $y=\sqrt{x}=x^{1/2}$

$y'=\frac{1}{2}x^{-1/2}=\frac{1}{2x^{1/2}}=\frac{1}{2\sqrt{x}}$

e) $y=\frac{1}{x}=x^{-1}$

$y'=-1\cdot x^{-2}=\frac{-1}{x^2}$

f) $f(x)=\frac{1}{x^3}=x^{-3}$

$f'(x)=-3x^{-4}=\frac{-3}{x^4}$

g) $s(t)=\frac{1}{\sqrt[3]{t}}=t^{-1/3}$

$s'(t)=-\frac{1}{3}t^{-4/3}=\frac{-1}{3t^{4/3}}$

h) $y=\frac{1}{x^{3/4}}=x^{-3/4}$

$y'=-\frac{3}{4}x^{-7/4}=-\frac{3}{4x^{7/4}}$

4) The constant multiple rule: If f is a differentiable function and c is a real number, then $\frac{d}{dx}[cf(x)]=cf'(x)$

Take the derivatives of the following. Use correct notation.

a) $y=\frac{2}{x^2}=2x^{-2}$

$y'=2\cdot -2x^{-3}=\frac{-4}{x^3}$

b) $f(x)=\frac{4x^3}{3}$

$f'(x)=\frac{4}{3}\cdot 3x^2=4x^2$

c) $s(t)=-t^5$

$s'(t)=-5t^4$

d) $y=4\sqrt{x}=4x^{1/2}$

$y'=4\cdot\frac{1}{2}x^{-1/2}=\frac{2}{x^{1/2}}=\frac{2}{\sqrt{x}}$

e) $y=\frac{-5}{3x^3}=\frac{-5}{3}x^{-3}$

$y'=\frac{-5}{3}\cdot -3x^{-4}$

$=\frac{5}{x^4}$

f) $f(x)=\frac{-5}{(3x)^3}=\frac{-5}{27x^3}$

$=\frac{-5}{27}x^{-3}$

$f'(x)=\frac{-5}{27}\cdot -3x^{-4}$

$=\frac{5}{9x^4}$

g) $s(t)=\frac{4}{\sqrt{t}}=4t^{-1/2}$

$s'(t)=4\cdot\frac{-1}{2}t^{-3/2}$

$=\frac{-2}{t^{3/2}}$

h) $y=\frac{-12}{\sqrt[3]{x^5}}=-12x^{-5/3}$

$y'=-12\cdot\frac{-5}{3}x^{-8/3}$

$=\frac{20}{x^{8/3}}$

18

5. The sum and difference rules. The derivative of a sum or difference is the sum or difference of the derivatives.

$$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x) \quad \text{and} \quad \frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$$

Take the derivatives of the following. Use correct notation.

a) $y = x^2 + 5x - 3$

$y' = 2x + 5\cdot 1 - 0$

$= 2x + 5$

b) $f(x) = x^4 - \frac{3}{2}x^3 + 2x^2 + x - 6$

$f'(x) = 4x^3 - \frac{3}{2}\cdot 3x^2 + 2\cdot 2x^1 + 1 - 0 = 4x^3 - \frac{9}{2}x^2 + 4x + 1$

c) $y = \frac{4}{x} - \frac{4}{x^2} + \frac{4}{x^3} = 4x^{-1} - 4x^{-2} + 4x^{-3}$

$y' = 4\cdot -1x^{-2} - 4\cdot -2x^{-3} + 4\cdot -3x^{-4}$

$= \frac{-4}{x^2} + \frac{8}{x^3} - \frac{12}{x^4}$

d) ~~$y = 6\sqrt{x}\left(\sqrt[3]{x} - 2x + 6\right)$~~

e) $f(x) = (2x-3)^2$

$= (2x-3)(2x-3)$

$= 4x^2 - 12x + 9$

$f'(x) = 4\cdot 2x^1 - 12\cdot 1 + 0$

$= 8x - 12$

f) ~~$y = (x^2 - x + 1)^2$~~

Higher-order Derivatives

Since the derivative of a function is also a function, we can take the derivative of it. This is called the second derivative which is again a function. So we can take the derivative of it, and so on.

Higher Order Derivatives		
Called	Prime Notation	Differential Notation
Original Function, or 0th derivative	$y = f(x) = f^{(0)}(x)$	$y = f(x)$
1st Derivative of f(x)	$y' = f'(x)$	$\frac{dy}{dx} = \frac{d}{dx}[y] = \frac{d}{dx}[f(x)]$
2nd Derivative of f(x)	$y'' = f''(x)$	$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}[y] = \frac{d^2}{dx^2}[f(x)]$
3rd Derivative of f(x)	$y''' = f'''(x)$	$\frac{d^3y}{dx^3} = \frac{d^3}{dx^3}[y] = \frac{d^3}{dx^3}[f(x)]$
4th Derivative of f(x)	$y^{(4)} = f^{(4)}(x)$	$\frac{d^4y}{dx^4} = \frac{d^4}{dx^4}[y] = \frac{d^4}{dx^4}[f(x)]$
5th Derivative of f(x)	$y^{(5)} = f^{(5)}(x)$	$\frac{d^5y}{dx^5} = \frac{d^5}{dx^5}[y] = \frac{d^5}{dx^5}[f(x)]$
...	...	...
nth Derivative of f(x)	$y^{(n)} = f^{(n)}(x)$	$\frac{d^ny}{dx^n} = \frac{d^n}{dx^n}[y] = \frac{d^n}{dx^n}[f(x)]$

Example #1: For $f(x) = 7x^3 - 8x^2 + 9x + 6$, find

a) $f'(x)$ b) $f''(x)$ c) $f'''(x)$ d) $f^{(4)}(x)$ e) $f^{(276)}(x)$

$f'(x) = 7\cdot 3x^2 - 8\cdot 2x^1 + 9\cdot 1 + 0$

$= 21x^2 - 16x + 9$

$f''(x) = 21\cdot 2x^1 - 16\cdot 1 + 0$

$= 42x - 16$

$f'''(x) = 42\cdot 1 - 0$

$= 42$

$f^{(4)}(x) = 0$

$f^{(276)}(x) = 0$

Example #2: For $f(x) = x^4$, find

a) $f'(x)$ b) $f''(x)$ c) $f'''(x)$

$4x^3$

$4\cdot 3x^2$

$12x^2$

$12\cdot 2x^1$

$24x$