Notes -- Binomial Distribution

When we have a number of independent trials in which there are two possible results (*success*=the event occurs or *failure*=the event does not occur), we have a binomial experiment.

The four major principles of a binomial distribution are:

1. 2 outcomes (success or failure)
2. fixed # of trials
3. probability of success is the same for each trial
4. trials are independent

Binomial Probability:

If X has a binomial distribution with *n* observations and probability *p* of success on each observation, the possible values of X are 0, 1,2,3,…,n. If *k* is any one of these values, then

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Expected Value = np

Many binomial calculations can be done on your calculator.

When looking for an exact probability we will use: **P(X=k)= binompdf (n, p, k)**
When looking for probabilities that are cumulative we will use: **P (X ≤k) = binomcdf (n, p, k)**

You must be careful when using binomcdf in your calculator! When you are working with cdf, your calculator will only read less than or less than or equal.

So, if are calculator only reads less than or less than or equal to, how can we find greater than or greater than or equal to?

When to use binompdf or binomcdf:

To find on the calculator: 2nd, vars, and scroll down

n = sample size   p = percent given   r = number of successes (number looking for)

- *Exact number, such as all, half, none, or a specific number*: binompdf(n,p,r)
- *No more than, at most, does not exceed*: binomcdf(n,p,r)
- *Less than or fewer than*: binomcdf(n,p,r-1)
- *At least, or more, no fewer than, not less than*: 1 – binomcdf(n,p,r-1)
- *More than*: 1 – binomcdf(n,p,r)

Example 1: Suppose you receive a shipment of five monkey-scooters. Each scooter has a 15% chance of not working. What is the probability that 3 scooters in your shipment will be defective?

Question: Why does this situation satisfy the binomial setting?

1. defective/not defective
2. 5 trials/scooters
3. p(defective) = .15
4. 1 defective scooter has no bearing on another scooter

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Label everything you know!

n = 5 p = .15 q= .85 x = 3

Let's start off by filling in a probability distribution table.

In order to determine the EXACT number for each probability we use Binomial pdf in our calculator.

P(X=k)= binompdf (5 , .15 , (varies)) x

X	0	1	2	3	4	5
P(X)	.444	.392	.138	.024	.002	7.594×10^{-5}

What is the probability 3 scooters in your shipment will be defective?

P(x = 3): .024

Using your chart, find the following and state the probability statements with each question:

a. The probability that 3 or more scooters are defective.
$p(x \geq 3) = p(3) + p(4) + p(5) = .026$ 1- binomcdf (5, .15, 3-1) 2

b. The probability that no more than 1 scooter is defective.
$p(x \leq 1) = p(0) + p(1) = .836$ binomcdf (5, .15, 1)

c. The probability that at most 4 are defective.
$p(x \leq 4) = p(0) + p(1) + p(2) + p(3) + p(4) = 1$ binomcdf (5, .15, 4)

d. The probability that fewer than 2 are defective.
$p(x < 2) = p(0) + p(1) = .836$ binomcdf (5, .15, 2-1)

e. What is the expected number of defective scooters in the shipment?
$5(.15) = .75$

__Example 2__: Suppose I am successful on 75% of my penalty shot attempts. My coach told me I had to make 9 attempts. Build a probability distribution table, then find the following probabilities by using your chart and using your calculator.

n = 9 p = .75 q= .25 x = varies

X	0	1	2	3	4	5	6	7	8	9
P(X)	3.815×10^{-6}	1.02997×10^{-4}	.001	.009	.039	.117	.234	.300	.225	.075

.000003815 .000102

expect 9(.75) = 6.75 to be made

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a. Find the probability that I make exactly 7 of those shots.

$p(x=7) = .300$ binompdf (9, .75, 7)

b. Find the probability that I make at least 7 of those shots.

$p(x \geq 7) = p(7) + p(8) + p(9) = .6$ 1−binomcdf (9, .75, 7−1)

c. Find the probability that I make no more than 3 of those shots.

$p(x \leq 3) = p(0) + p(1) + p(2) + p(3) = .010$ binomcdf (9, .75, 3)

d. Find the probability that I make at most 5 of those shots.

$p(x \leq 5) = p(0) + p(1) + p(2) + p(3) + p(4) + p(5) = .166$

binomcdf (9, .75, 5)

What if… the number of attempts is 50??? Find the following probabilities.

a. Find the probability that I make exactly 40 of those shots.

$p(x=40) =$ binompdf (50, .75, 40) $= .099$

b. Find the probability that I make at least 40 of those shots.

$p(x \geq 40) = 1 -$ binomcdf (50, .75, 39) $= .262$

c. Find the probability that I make no more than 27 of those shots.

$p(x \leq 27) =$ binomcdf (50, .75, 27) $= .001$

d. Find the probability that I make at most 18 of those shots.

$p(x \leq 18) =$ binomcdf (50, .75, 18) $= 6.717 \times 10^{-9}$

.0000000006717

expect $50(.75) = 37.5$

Statistics – Binomials

(When to use: binompdf or binomcdf)

To find on the calculator: 2nd, vars, and scroll down

n = sample size
p = percent given
r = number of successes (number looking for)

1. Exact number, such as all, half, none, or a specific number: binompdf(n,p,r)
2. No more than, at most, does not exceed: binomcdf(n,p,r)
3. Less than or fewer then: binomcdf(n,p,r-1)
4. At least, or more, no fewer than , not less than: 1 – binomcdf(n,p,r-1)
5. More than: 1 – binomcdf(n,p,r)
6. Between two numbers, where a is the small number and b is the larger number: binomcdf(n,p,b) – binomcdf(n,p,a-1)
7. Looking for n, to be at least P% sure: 1 – binomcdf(n,p,r-1) [Guess numbers for n]
8. Normal approximation to the binomial distribution: Subtract 0.5 from lower bound, add 0.5 to upper bound

p = success
$q = 1 - p$
$\mu = np$
$\sigma = \sqrt{npq}$