Converting Degrees to Radians and Radians to Degrees

$C = \pi \cdot d = 2\pi r_{rad.}$ $\quad C = 360^\circ$

$2\pi r = 360^\circ$

$r = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi}$

$\frac{2\pi r}{360^\circ} = \frac{360^\circ}{360^\circ}$

$\frac{\pi}{180^\circ} \cdot r = 1^\circ$

degrees $\xrightarrow{\times \frac{\pi}{180}}$ radians $\xrightarrow{\times \frac{180}{\pi}}$ degrees

Example 1 Convert 40° to radians.

$40^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{9}$ radians

Example 2 Convert $\frac{7\pi}{6}$ to degrees.

$\frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} = 210^\circ$

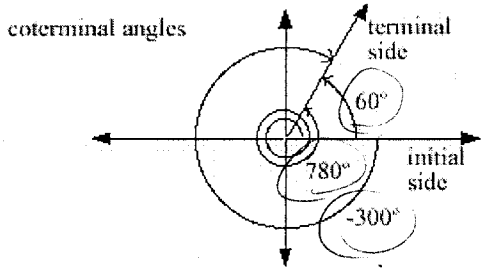
Example 3 Use a calculator to convert 16° to radians.

$16^\circ \cdot \frac{\pi}{180^\circ} = .279$

Example 4 Use a calculator to convert $\frac{3\pi}{8}$ to degrees.

$\frac{3\pi}{8} \cdot \frac{180^\circ}{\pi} = 67.5^\circ$

coterminal angles are angles in standard position (angles with the initial side on the positive x-axis) that have a common terminal side.



Degree measure	Radian measure
Formula $\theta + 360n$	Formula $\theta + 2n\pi$

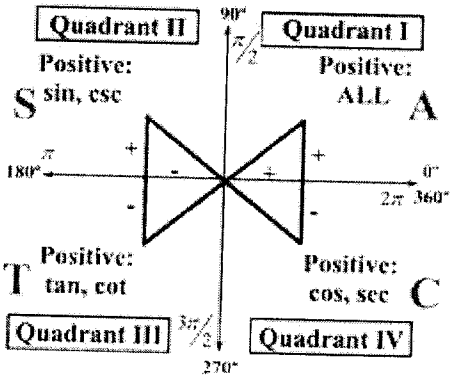
Use positive integer values for n to find the positve coterminals and negative integer values to find negative coterminal angles.

Example 5 Find two other angles, one negative and one positive, which are coterminal to θ.

A. $\theta = 85°$ B. $\theta = -337°$ C. $\theta = \frac{2\pi}{3}$ D. $\theta = -3\pi$

$85° + 360° = 445°$ $-337° + 360° = 23°$ $\frac{2\pi}{3} + 2\pi = \frac{8}{3}\pi$ $-3\pi + 2\pi = -\pi$

$85° - 360° = -275°$ $-337° - 360° = -697°$ $\frac{2}{3}\pi - 2\pi = -\frac{4\pi}{3}$ $-3\pi + 2(2\pi) = \pi$



Example 6 Find the exact value of each of the six trig functions of the given angle.

A. $\theta = \frac{2\pi}{3}$ $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ quad 2 (x, y)

$\cos\theta = -\frac{1}{2}$ $\sec\theta = -2$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\csc\theta = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot -\frac{2}{1} = -\sqrt{3}$ $\cot\theta = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

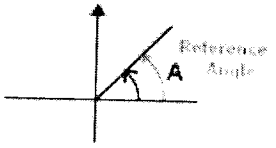
B. $\theta = 675°$ coterminal with $315°$ quad 4 $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\cos\theta = \frac{\sqrt{2}}{2}$ $\sec\theta = \frac{2 \cdot \sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

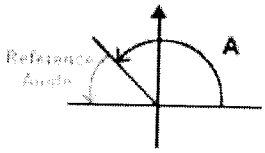
$\sin\theta = -\frac{\sqrt{2}}{2}$ $\csc\theta = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

$\tan\theta = -1$ $\cot\theta = -1$

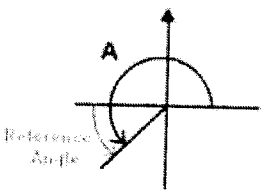
reference angle -- the angle that the given angle makes with the *x*-axis. Regardless of where the angle ends (that is, regardless of the location of the terminal side of the angle), the reference angle measures the closest distance of that terminal side to the *x*-axis.



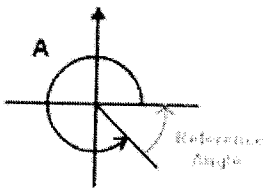
1st Quadrant



2nd Quadrant



3rd Quadrant



4th Quadrant