AP Calculus BC Review Notes Curve Analysis and Optimization

You should be able to . . .

- **Analyze a function for increasing/decreasing, relative/absolute extrema, concavity, points of inflection, etc.**

Situation	Indicates
$f'(c) > 0$	f increasing at c
$f'(c) < 0$	f decreasing at c
$f'(c) = 0$	horizontal tangent at c
$f'(c) = 0,\ f'(c^-) < 0,\ f'(c^+) > 0$	relative minimum at c
$f'(c) = 0,\ f'(c^-) > 0,\ f'(c^+) < 0$	relative maximum at c
$f'(c) = 0,\ f''(c) > 0$	relative minimum at c
$f'(c) = 0,\ f''(c) < 0$	relative maximum at c
$f'(c) = 0,\ f''(c) = 0$	*further investigation required*
$f''(c) > 0$	concave upward
$f''(c) < 0$	concave downward
$f''(c) = 0$	*further investigation required*
$f''(c) = 0,\ f''(c^-) < 0,\ f''(c^+) > 0$	point of inflection
$f''(c) = 0,\ f''(c^-) > 0,\ f''(c^+) < 0$	point of inflection
$f(c)$ exists, $f'(c)$ does not exist	possibly a vertical tangent; possibly an absolute max. or min.

Ex 1) For each function, analyze where the function is increasing or decreasing, concavity, extrema and points of inflection.

A. $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

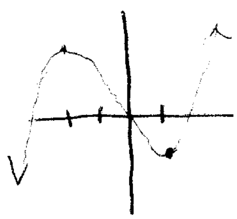
$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$

$x = -2 \quad x = 1$ critical values

$f'(x)$: $+ \;|\; - \;|\; +$

x: -2, 1



$f'(x) + \rightarrow f$ incr $(-\infty, -2) \cup (1, \infty)$

$f'(x) - \rightarrow f$ decr $(-2, 1)$

rel max at $x = -2$

rel min at $x = 1$

$f''(x) = 12x + 6$

$6(2x+1) = 0$

$x = -\frac{1}{2}$

$f''(x)$: $- \;|\; +$

x: $-1/2$

f'' neg $\rightarrow f$ concave down $(-\infty, -1/2)$

f'' pos $\rightarrow f$ concave up $(-1/2, \infty)$

POI when $x = -1/2$

B. $f(x) = \frac{1}{(x-2)^2} = (x-2)^{-2}$

$f'(x) = -2(x-2)^{-3}(1) = \frac{-2}{(x-2)^3}$

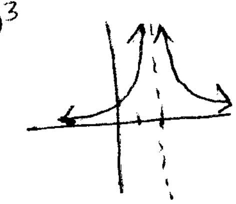
f' undef. for $x = 2$

$f'(x)$: $+ \;|\; -$

x: 2

f incr $(-\infty, 2)$ f decr $(2, \infty)$

no rel. max/min



$f''(x) = 6(x-2)^{-4}(1) = \frac{6}{(x-2)^4}$

f'' undef at $x = 2$

$f''(x)$: $+ \;|\; +$

x: 2

no POI

concave up $(-\infty, 2) \cup (2, \infty)$

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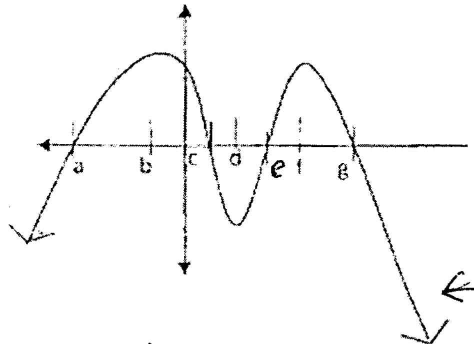
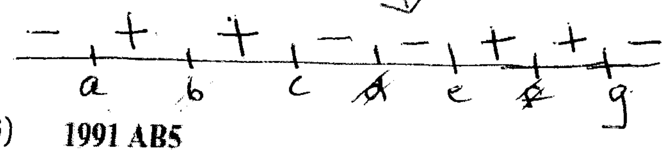
Ex 2) Given $f'(x)$, find where $f(x)$ is increasing or decreasing. State the values of x where extrema occur.





f incr where f′ +
(a,c) (e,g)

f decr where f′ −
$(-\infty,a)$ (c,e) (g,∞).

rel min at x=a, x=e
rel max at x=c, x=g

Ex 3) **1991 AB5**

refl over y-axis

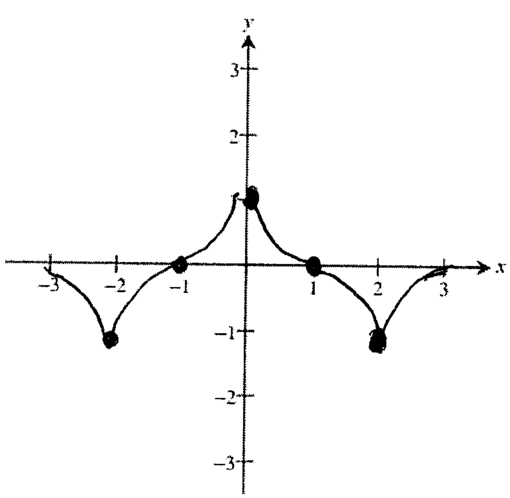
Let f be a function that is even and continuous on the closed interval $[-3,3]$. The function f and its derivatives have the properties indicated in the table below.

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$
$f(x)$	1	Positive	0	Negative	−1	Negative
$f'(x)$	Undefined	Negative	0	Negative	Undefined	Positive
$f''(x)$	Undefined	Positive	0	Negative	Undefined	Negative

(a) Find the x-coordinate of each point at which f attains an absolute maximum value or an absolute minimum value. For each x-coordinate you give, state whether f attains an absolute maximum or an absolute minimum.

(b) Find the x-coordinate of each point of inflection on the graph of f. Justify your answer.

(c) In the xy-plane provided below, sketch the graph of a function with all the given characteristics of f.



a)

x	y
0	1
decr	
1	less than 1
2	−1
incr 3	neg

abs. max at x=0
abs. min at x=2
abs. min at x=−2 (even)

b) POI at x=1 f″ changes from pos to neg

POI at x=−1 f is even

Ex 4) Find all points of the absolute extrema over the given interval: $y = x^3 + 6x^2 + 9x + 3$; $[-4, 0]$

$y' = 3x^2 + 12x + 9$

$3(x^2 + 4x + 3) = 0$

$3(x+3)(x+1) = 0$

$x = -3, \ x = -1$

$+ \quad | \quad - \quad | \quad +$

$\quad -3 \qquad -1$

$x = -3$ rel max

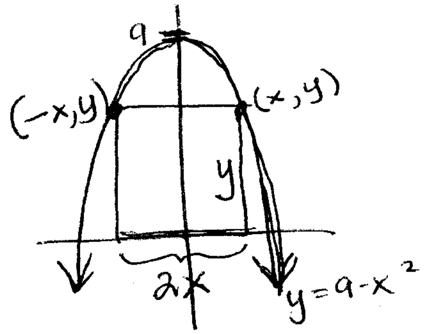
$x = -1$ rel min

x	y
-4	-1
-3	3
-1	-1
0	3

abs. min:
$(-4,-1)$ and $(-1,-1)$

abs. max:
$(-3,3)(0,3)$

Ex 5) Find the dimensions of the rectangle of the largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 9 - x^2$.



maximize area

$A = 2x \cdot y = 2x(9 - x^2) = 18x - 2x^3$

$A' = 18 - 6x^2 = 0$

$-6x^2 = -18$

$x^2 = 3$

$x = \pm\sqrt{3}$

$+ \qquad -$

$\cancel{-\sqrt{3}} \qquad \sqrt{3}$

max

$x = \sqrt{3}$

$y = 9 - (\sqrt{3})^2 = 6$

$\boxed{2\sqrt{3} \text{ by } 6}$