$f(x) = \dfrac{x^2+1}{x^2-4}$ Analyze and Sketch.

x-int. $0 = \dfrac{x^2+1}{x^2-4}$

$x^2+1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1} = \pm i$

none

y-int. $f(0) = \dfrac{0^2+1}{0^2-4} = -1/4$

$(0, -1/4)$

asymptotes $f(x) = \dfrac{x^2+1}{(x-2)(x+2)}$ vert. asympt. $x=2$, $x=-2$

$\lim\limits_{x\to\pm\infty} \dfrac{x^2+1}{x^2-4} = 1$ horiz. asympt. $y=1$

$f'(x) = \dfrac{(x^2-4)(2x) - (x^2+1)(2x)}{(x^2-4)^2} = \dfrac{2x^3-8x-2x^3-2x}{(x^2-4)^2} = \dfrac{-10x}{(x^2-4)^2}$

Critical values at $x = 0, 2, -2$

f': $+$ | $+$ | $-$ | $-$ (at -2, 0, 2)

f incr $(-\infty, -2)$ $(-2, 0)$

f decr $(0, 2)$ $(2, \infty)$

rel max $(0, -1/4)$

$$f'': \frac{(x^2-4)^{1}(-10) - (-10x)\,2(x^2-4)(2x)}{(x^2-4)^{3}}$$

$$\frac{-10x^2+40+40x^2}{(x^2-4)^3} = \frac{30x^2+40}{(x^2-4)^3}$$

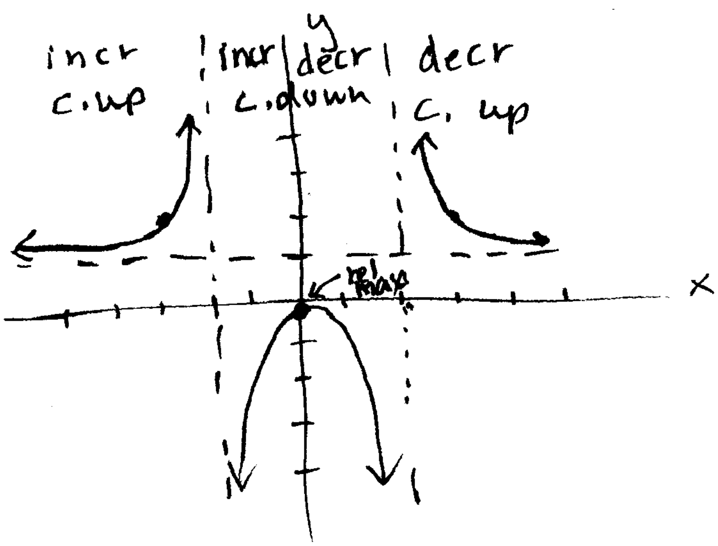
undef at $x = 2, -2$

$+ \mid - \mid +$ (at -2, 2)

f conc. up $(-\infty, -2)$, $(2, \infty)$

f conc. down $(-2, 2)$

no POI



x	$f(x)$
-3	$\frac{10}{5} = 2$
3	$\frac{10}{5} = 2$