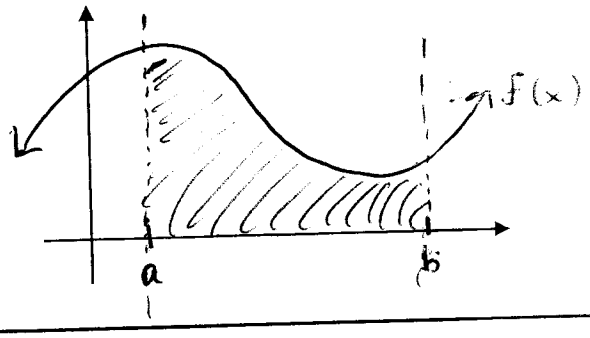
Calculus AB

Notes: Definite Integrals

Definite Integral

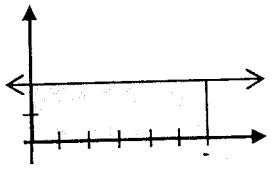
If f is continuous and non-negative on the closed interval $[a,b]$, then the area of the region bounded by the graph of f, the x-axis and the vertical lines $x=a$ and $x=b$ is given by:

$$\text{Area} = \int_a^b f(x)\,dx$$



Ex 1: Find the area.

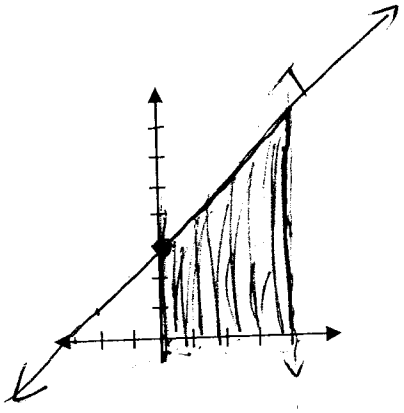
A. $\int_0^6 2dx$



rect

$6(2) = 12$

B. $\int_0^4 (x+3)dx$



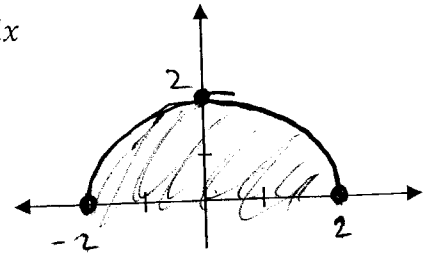
trap

$\frac{1}{2}(4)(3+7) = 20$

C. $\int_{-2}^{2} \sqrt{4-x^2}\,dx$

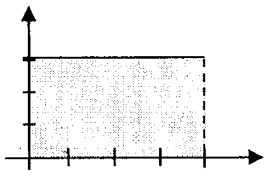
Semicircle

$\frac{1}{2}\pi(2)^2 = 2\pi$



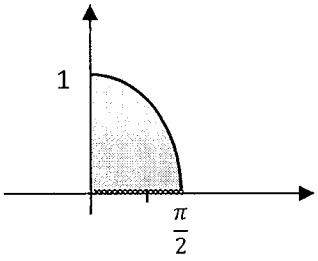
Ex 2: Write an integral that represents each of the following areas.

A.



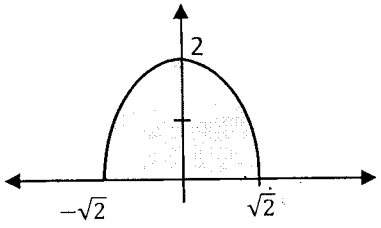
$$\int_0^4 3\,dx$$

B.



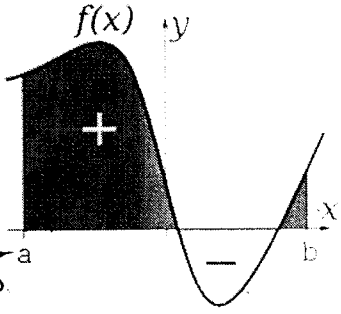
$$\int_0^{\pi/2} \cos x\,dx$$

C.



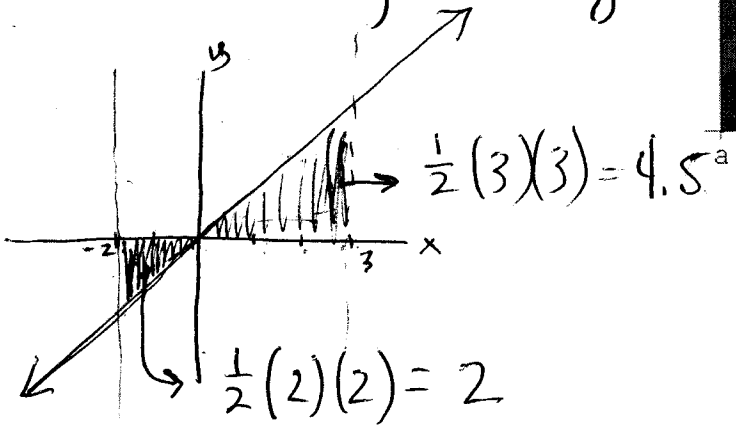
$$\int_{-\sqrt{2}}^{\sqrt{2}} (-x^2+2)\,dx$$

➢ If a graph falls below the x-axis that portion is considered "negative area" when evaluating an integral



Ex 3: Evaluate.

$$\int_{-2}^{3} x\,dx$$



$4.5 - 2$

$\boxed{2.5}$

8

Properties of Integrals

1. $\int_a^a f(x)dx = 0$

2. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

4. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$

5. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

Ex 4: Given $\int_2^4 x^3dx = 60$, $\int_2^4 xdx = 6$, & $\int_2^4 dx = 2$

find:

A. $\int_2^2 x^3dx = 0$

B. $\int_2^4 15dx = 15\int_2^4 dx = 15 \cdot 2 = 30$

C. $\int_2^4 (x^3+4)dx = \int_2^4 x^3dx + \int_2^4 4dx$

$= 60 + 4(2) = 68$

D. $\int_2^4 (6+2x-x^3)dx$

$= \int_2^4 6dx + \int_2^4 2xdx - \int_2^4 x^3dx$

$= 6(2) + 2(6) - 60 = -36$