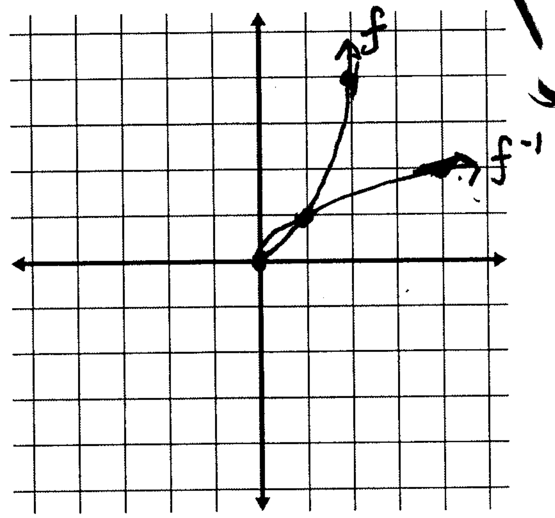
Developing the Idea of the Derivative of an Inverse . . .



A. Graph $f(x)=x^2$ for $x \geq 0$.

range $y \geq 0$

B. Find the slope of the tangent line at x = 2.

$f'(x) = 2x$

$f'(2) = 4$

C. Find the inverse of $f(x)$.

$x = y^2$

$y = \pm\sqrt{x}$ $\quad f^{-1}(x) = \sqrt{x} \quad x \geq 0$

D. Find the derivative of the inverse.

$\left(f^{-1}(x)\right)' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

E. Find the slope of the tangent line at x = 4.

$\left(f^{-1}(4)\right)' = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

F. Compare the value of $\left(f^{-1}\right)'(4)$ to that of $f'(2)$. What is the relationship?

reciprocals

❖ The derivative of an inverse evaluated at $f(a)$ (y-value on f) is equal to the reciprocal of $f'(x)$ evaluated at $x = a$.

❖ If g is the inverse of f, then $g'(x) = \dfrac{1}{f'(g(x))} = \dfrac{1}{f'(f^{-1}(x))}$

AP Calculus AB

Notes – Derivatives of Inverse Functions

Review: *How do you find the inverse of a function?* Switch x's and y's

What is a one-to-one function? HLT test -- inverse is a function

Let f be a one-to-one differentiable function with inverse function f^{-1}.
If $y = f^{-1}(x)$ so that $x = f(y)$, then $dy/dx = 1/(dx/dy)$ with $dy/dx \neq 0$.

Ex1) If $f(x) = x^3 + 2x - 10$, find $(f^{-1})'(x)$

$x = y^3 + 2y - 10$

$1 = 3y^2 \frac{dy}{dx} + 2\frac{dy}{dx}$

$1 = \frac{dy}{dx}(3y^2 + 2)$

$\frac{dy}{dx} = \frac{1}{3y^2+2}$

Ex2) If $f(x) = 2x^5 + x^3 + 1$, find **(a)** $f(1)$ & $f'(1)$ **(b)** $(f^{-1})(4)$ & $(f^{-1})'(4)$

a) $f(1) = 2(1)^5 + (1)^3 + 1 = 4$

$f'(x) = 10x^4 + 3x^2$

$f'(1) = 10 + 3 = 13$

b) $f: (1,4)$

$f^{-1}: (4,1)$

$f^{-1}(4) = 1$

$(f^{-1})'(4) = \frac{1}{13}$

Ex3) If $f(x) = 5x^3 + x + 8$, find $(f^{-1})'(8)$.

$5x^3 + x + 8 = 8$

$5x^3 + x = 0$

$x(5x^2+1) = 0$

$x = 0 \quad 5x^2 + 1 = 0$

$f^{-1}: (8, 0)$

$f: (0, 8)$

$f'(x) = 15x^2 + 1$

$f'(0) = 1$

$(f^{-1})'(8) =$ recip of $) = 1$

Ex4) If $f(2) = -3$, $f'(2) = 3/4$, and $g(x) = f^{-1}(x)$, what is the equation of the tangent line to $g(x)$ at $x = -3$?

point $(-3, g(-3)) \rightarrow (-3, 2)$

slope $g'(-3) = \frac{4}{3}$

$f: (2,-3)$

$g: (-3, 2)$

$f'(2) = \frac{3}{4}$

$g'(-3) = \frac{4}{3}$

$y - 2 = \frac{4}{3}(x+3)$

Ex5) Let $h(x) = xg(x)$, where $g(x) = f^{-1}(x)$. Use the table of values below to find $h'(5)$.

x	$f(x)$	$f'(x)$
2	4	−1
3	5	2
5	1	3

(a) ½ (b) 2.5 (c) 3 (d) 4.667 (e) 5.5

$h'(x) = x \cdot g'(x) + g(x) \cdot 1$

$h'(5) = 5 \cdot g'(5) + g(5)$

$= 5 \cdot \frac{1}{2} + 3$

$= 5.5$

$g: (5, 3)$

$f: (3, 5)$

$f'(3) = 2$

$g'(5) = \frac{1}{2}$

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$$x = 2y^5 + y^3 + 1$$

$$1 = 10y^4\frac{dy}{dx} + 3y^2\frac{dy}{dx}$$

$$1 = \frac{dy}{dx}\left(10y^4 + 3y^2\right)$$

$$\frac{dy}{dx} = \frac{1}{10y^4 + 3y^2}$$

when $x = 4$, $y = 1$ $\quad \left.\frac{dy}{dx}\right|_{y=1} = \frac{1}{10(1)^4 + 3(1)^2} = \frac{1}{13}$

$$x = 5y^3 + y + 8$$

$$1 = 15y^2\frac{dy}{dx} + 1\frac{dy}{dx}$$

$$1 = \frac{dy}{dx}\left(15y^2 + 1\right)$$

$$\frac{dy}{dx} = \left.\frac{1}{15y^2 + 1}\right|_{y=0} = 1$$