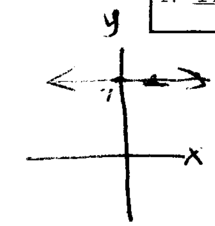
Notes--Basic Derivative Rules

Taking derivatives is a a process that is vital in calculus. In order to take derivatives, there are rules that will mak the process simpler than having to use the definition of the derivative.

1. The constant rule: The derivative of a constant function is 0. That is, if c is a real number, then $\frac{d}{dx}[c] = 0$.



a) $y = 7$

$y' = 0$

b) $f(x) = 0$

$f'(x) = 0$

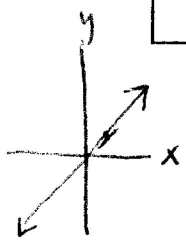
c) $s(t) = -8$

$s'(t) = 0$

d) $y = \pi^3$

$\frac{dy}{dx} = 0$

2. The single variable rule: The derivative of x is 1. $\frac{d}{dx}[x] = 1$. This is consistent with the fact that the slope of the line $y = x$ is 1.



a) $y = x$

$y' = 1$

b) $f(x) = x$

$f'(x) = 1$

c) $s(t) = t$

$s'(t) = 1$

3. The power rule: If n is a rational number then the function x^n is differentiable and $\frac{d}{dx}[x^n] = nx^{n-1}$.

Take the derivatives of the following. Use correct notation.

a) $y = x^2$

$y' = 2 \cdot x^1$

b) $f(x) = x^6$

$f'(x) = 6x^5$

c) $s(t) = t^{30}$

$s'(t) = 30t^{29}$

d) $y = \sqrt{x} = x^{\frac{1}{2}}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{1/2}}$

e) $y = \frac{1}{x} = x^{-1}$

$y' = -1 \cdot x^{-2} = \frac{-1}{x^2}$

f) $f(x) = \frac{1}{x^3} = x^{-3}$

$f'(x) = -3x^{-4} = \frac{-3}{x^4}$

g) $s(t) = \frac{1}{\sqrt[3]{t}} = t^{-\frac{1}{3}}$

$s'(t) = -\frac{1}{3}t^{-4/3} = \frac{-1}{3t^{4/3}}$

h) $y = \frac{1}{x^{3/4}} = x^{-3/4} = \frac{1}{2\sqrt{x}}$

$y' = -\frac{3}{4}x^{-7/4} = \frac{-3}{4x^{7/4}}$

4) The constant multiple rule: If f is a differentiable function and c is a real number, then $\frac{d}{dx}[cf(x)] = cf'(x)$

Take the derivatives of the following. Use correct notation.

a) $y = \frac{2}{x^2} = 2 \cdot x^{-2}$

$y' = 2 \cdot -2x^{-3} = \frac{-4}{x^3}$

b) $f(x) = \frac{4x^3}{3}$

$f'(x) = \frac{4}{3} \cdot 3x^2 = 4x^2$

c) $s(t) = -t^5$

$s'(t) = -1 \cdot 5t^4 = -5t^4$

d) $y = 4\sqrt{x} = 4x^{1/2}$

$y' = 4 \cdot \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{2}{\sqrt{x}}$

e) $y = \frac{-5}{3x^3} = -\frac{5}{3}x^{-3}$

$y' = -\frac{5}{3} \cdot -3x^{-4}$

$= \frac{5}{x^4}$

f) $f(x) = \frac{-5}{(3x)^3} = \frac{-5}{27x^3}$

$= -\frac{5}{27} \cdot x^{-3}$

$f'(x) = -\frac{5}{27} \cdot -3x^{-4}$

$= \frac{5}{9x^4}$

g) $s(t) = \frac{4}{\sqrt{t}} = 4t^{-1/2}$

$s'(t) = 4 \cdot -\frac{1}{2}t^{-3/2}$

$= \frac{-2}{t^{3/2}}$

h) $y = \frac{-12}{\sqrt[3]{x^5}} = -12x^{-\frac{5}{3}}$

$y' = -12 \cdot -\frac{5}{3}x^{-\frac{8}{3}}$

$= \frac{20}{x^{8/3}}$

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5. The sum and difference rules. The derivative of a sum or difference is the sum or difference of the derivatives.

$$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x) \quad \text{and} \quad \frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$$

Take the derivatives of the following. Use correct notation.

a) $y = x^2 + 5x - 3$

$y' = 2x^1 + 5\cdot 1 - 0 = 2x+5$

b) $f(x) = x^4 - \frac{3}{2}x^3 + 2x^2 + x - 6$

$f'(x) = 4\cdot x^3 - \frac{3}{2}\cdot 3x^2 + 2\cdot 2x^1 + 1 - 0$

$= 4x^3 - \frac{9}{2}x^2 + 4x + 1$

c) $y = \frac{4}{x} - \frac{4}{x^2} + \frac{4}{x^3} = 4x^{-1} - 4x^{-2} + 4x^{-3}$

$y' = 4\cdot -1x^{-2} - 4\cdot -2x^{-3} + 4\cdot -3x^{-4}$

$= \frac{-4}{x^2} + \frac{8}{x^3} - \frac{12}{x^4}$

d) $f(x) = (2x-3)^2$

$f(x) = 4x^2 - 12x + 9$

$f'(x) = 4\cdot 2x^1 - 12\cdot 1 + 0$

$= 8x - 12$

Higher-order Derivatives

Since the derivative of a function is also a function, we can take the derivative of it. This is called the second derivative which is again a function. So we can take the derivative of it, and so on.

Higher Order Derivatives		
Called	Prime Notation	Differential Notation
Original Function, or 0^{th} derivative	$y = f(x) = f^{(0)}(x)$	$y = f(x)$
1^{st} Derivative of $f(x)$	$y' = f'(x)$	$\frac{dy}{dx} = \frac{d}{dx}[y] = \frac{d}{dx}[f(x)]$
2^{nd} Derivative of $f(x)$	$y'' = f''(x)$	$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}[y] = \frac{d^2}{dx^2}[f(x)]$
3^{rd} Derivative of $f(x)$	$y''' = f'''(x)$	$\frac{d^3y}{dx^3} = \frac{d^3}{dx^3}[y] = \frac{d^3}{dx^3}[f(x)]$
4^{th} Derivative of $f(x)$	$y^{(4)} = f^{(4)}(x)$	$\frac{d^4y}{dx^4} = \frac{d^4}{dx^4}[y] = \frac{d^4}{dx^4}[f(x)]$
5^{th} Derivative of $f(x)$	$y^{(5)} = f^{(5)}(x)$	$\frac{d^5y}{dx^5} = \frac{d^5}{dx^5}[y] = \frac{d^5}{dx^5}[f(x)]$
...	...	...
n^{th} Derivative of $f(x)$	$y^{(n)} = f^{(n)}(x)$	$\frac{d^ny}{dx^n} = \frac{d^n}{dx^n}[y] = \frac{d^n}{dx^n}[f(x)]$

Example #1: For $f(x) = 7x^3 - 8x^2 + 9x + 6$, find

a) $f'(x)$ b) $f''(x)$ c) $f'''(x)$ d) $f^{(4)}(x) = 0$ e) $f^{(276)}(x) = 0$

$f'(x) = 7\cdot 3x^2 - 8\cdot 2x^1 + 9\cdot 1 + 0$

$= 21x^2 - 16x + 9$

$f''(x) = 21\cdot 2x^1 - 16\cdot 1 + 0$

$= 42x - 16$

$f'''(x) = 42\cdot 1 - 0 = 42$

Example #2: For $f(x) = x^{-1}$, find

a) $f'(x)$ b) $f''(x)$ c) $f'''(x)$

$-1\cdot x^{-2}$

$-1\cdot -2x^{-3}$

$2x^{-3}$

$2\cdot -3x^{-4}$

$-6x^{-4}$