Diff. Eqns Day 2

Ex1 Find the general soln.

(A) $\dfrac{dy}{dx} = \dfrac{x^2+2}{3y^2}$

$$\int y^2\,dy = \int \frac{x^2+2}{3}\,dx$$

$$\frac{1}{3}y^3 = \frac{1}{9}x^3 + \frac{2}{3}x + C$$

$$y^3 = \frac{1}{3}x^3 + 2x + c$$

$$y = \sqrt[3]{\frac{1}{3}x^3 + 2x + C}$$

(B) $yy' - 2e^x = 0$

$$y\frac{dy}{dx} = 2e^x$$

$$\int y\,dy = \int 2e^x\,dx$$

$$\frac{1}{2}y^2 = 2e^x + C$$

$$y^2 = 4e^x + c$$

$$y = \pm\sqrt{4e^x + c}$$

EX2 Given $xy\,dx + e^{-x^2}(y^2-1)\,dy = 0$

Find the particular soln. if $y(0) = 1$.

$$e^{-x^2}(y^2-1)\,dy = -xy\,dx$$

$y - \frac{1}{y}$

$$\int \frac{y^2-1}{y}\,dy = \frac{-x}{e^{-x^2}}\,dx = \int\left(-xe^{x^2}\right)dx$$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\int e^u\,du$

$-\frac{1}{2}e^u$

$$\frac{1}{2}y^2 - \ln|y| = -\frac{1}{2}e^{x^2} + C$$

$$\frac{1}{2}(1)^2 - \ln|1| = -\frac{1}{2}e^{0^2} + C$$

$$\frac{1}{2} - 0 = -\frac{1}{2} + C$$

$$1 = C$$

$$\frac{1}{2}y^2 - \ln|y| = -\frac{1}{2}e^{x^2} + 1$$

$$y^2 - \underbrace{2\ln|y|}_{\ln y^2} = -e^{x^2} + 2$$

EX3 Given $y\sqrt{1-x^2}\,y' - x\sqrt{1-y^2} = 0$ and $(0,1)$

$$y\sqrt{1-x^2}\,\frac{dy}{dx} = x\sqrt{1-y^2}$$

$$\int \frac{y}{\sqrt{1-y^2}}\,dy = \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$u = 1-y^2$

$\frac{du}{dy} = -2y$

$-\frac{1}{2}\,du = y\,dy$

$-\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\int u^{-1/2}\,du$

$-1\,u^{1/2}$

$$-\sqrt{1-y^2} = -\sqrt{1-x^2} + C$$

$$-\sqrt{1-1^2} = -\sqrt{1-0^2} + C$$

$$0 = -1 + C$$

$$C = 1$$

$$-\sqrt{1-y^2} = -\sqrt{1-x^2} + 1$$

$$\left(\sqrt{1-y^2}\right)^2 = \left(\sqrt{1-x^2} - 1\right)^2$$

$$1-y^2 = \left(\sqrt{1-x^2} - 1\right)^2$$

$$-y^2 = \left(\sqrt{1-x^2} - 1\right)^2 - 1$$

$$y^2 = -\left(\sqrt{1-x^2} - 1\right)^2 + 1$$

$$y = \pm\sqrt{-\left(\sqrt{1-x^2} - 1\right)^2 + 1}$$

$$y = \sqrt{-\left(\sqrt{1-x^2} - 1\right)^2 + 1}$$