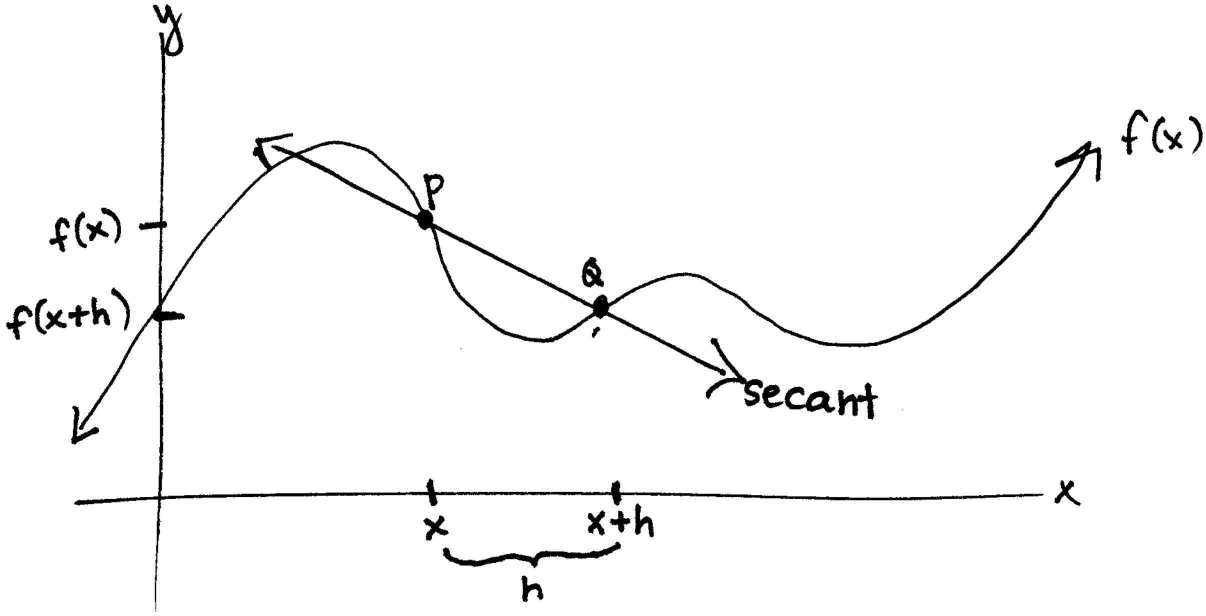

$P\ (x, f(x)) \quad Q\ (x+h, f(x+h))$

$$\text{slope of } \overleftrightarrow{PQ} = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h - x} = \frac{f(x+h)-f(x)}{h}$$

slope of secant
difference quotient
average rate of change

move Q closer to P $\Rightarrow$ $h \to 0$

Slope of tangent line $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

derivative
$f'(x)$
instantaneous rate of change

EX1 $f(x) = -3x^2$

a. Find the average rate of change for $[0,2]$.

b. Find the eqn. of the tangent to the curve at $x=2$.

c. Find the instantaneous rate of change at $x=1.5$.

a. $(0, f(0)) = (0,0) \qquad (2, f(2)) = (2,-12)$

$$m = \frac{-12-0}{2-0} = -6$$

b. point $(2,-12)$

Slope $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{-3(x+h)^2 - (-3x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-3x^2} - 6hx - 3h^2 + \cancel{3x^2}}{h}$$

$$= \lim_{h\to 0} (-6x - 3h)$$

$= -6x$ general

when $x=2$, $m = -6(2) = -12$

point-slope form $y - y_1 = m(x - x_1)$

$$\boxed{y+12 = -12(x-2)}$$

c. $f'(x) = -6x \quad f'(1.5) = -6(1.5) = -9$

EX2 $f(x) = 3x^2 - 6x$

a. Find the avg. r.o.c. for $[1,4]$

b. Find the slope of the secant for the pts. in part A.

c. Find the instan. r.o.c. for $x=2$.

d. Find the eqn of the tangent line at $x=2$

e. Find the eqn of the normal line at $x=2$.
$\hookrightarrow \perp$ to tangent line

a. $(1,-3)$ $(4,24)$

$$\text{slope} = \frac{24-(-3)}{4-1} = \frac{27}{3} = 9$$

b. 9 (see part A)

c. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x+h)^2 - 6(x+h) - (3x^2-6x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6hx + 3h^2 - \cancel{6x} - 6h - \cancel{3x^2} + \cancel{6x}}{h}$$

$$= \lim_{h\to 0} (6x + 3h - 6)$$

$$= 6x - 6 \quad \text{general}$$

$$f'(2) = 6(2) - 6 = 6$$

d. point $(2, f(2)) = (2, 0)$

slope $f'(x) = 6x - 6$

$f'(2) = 6$

$y - 0 = 6(x - 2)$

e. point $(2, 0)$

slope $-\frac{1}{6}$

$y - 0 = -\frac{1}{6}(x - 2)$

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alternate form of derivative

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$ "a" is a real #

<u>EX 3</u> $f(x) = x^3 - x$. Find $f'(2)$.

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2} \frac{x^3 - x - 6}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2 + 2x + 3)}{\cancel{x-2}}$$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -1 & -6 \\ & \downarrow & 2 & 4 & 6 \\ \hline & 1 & 2 & 3 & 0 \end{array}$$

$= 4 + 4 + 3$

$= 11$

$f'(x)$ $\frac{dy}{dx}$ y' $\frac{d(f(x))}{dx}$