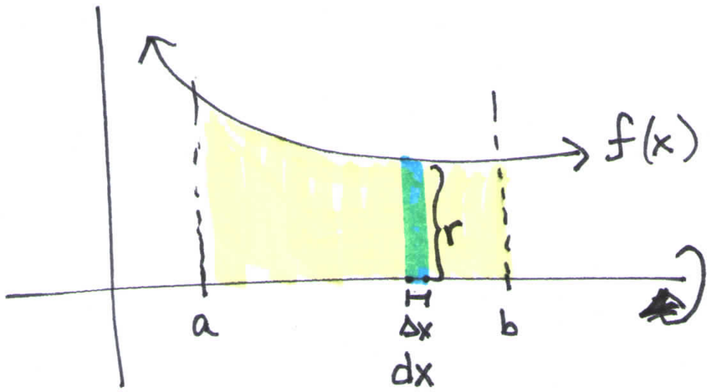
Volume – the Disk Method



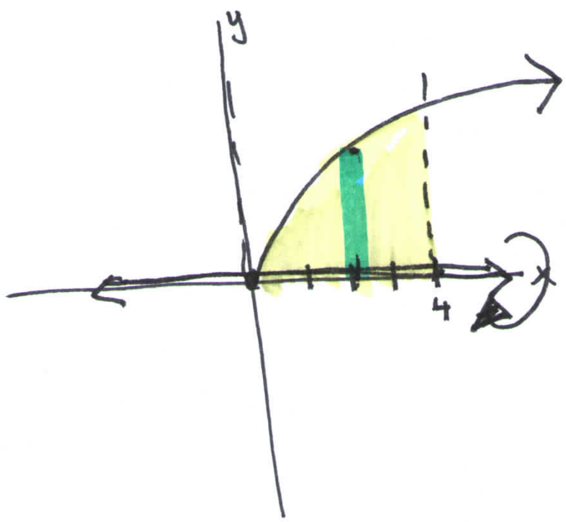
$$V = \pi \int_a^b [f(x)]^2 \, dx$$

$$V = \pi r^2 h$$

- use when the region is adjacent to the axis of revolution
- the rectangle is $\perp$ to the axis of revolution and $\perp$ to the axis of integration

Find the volume.

1. $y = \sqrt{x}$, $y = 0$, $0 \le x \le 4$
   revolve around the x-axis



$$V = \pi \int_0^4 (\sqrt{x})^2 \, dx$$

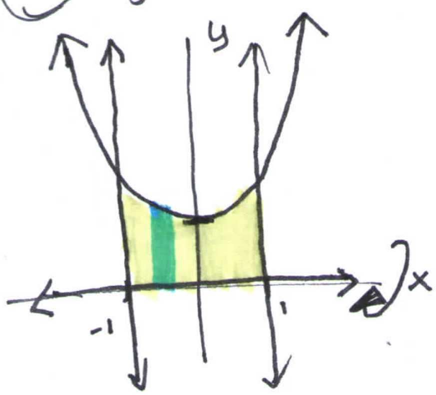
$$= \pi \int_0^4 x \, dx$$

$$= \pi \left(\tfrac{1}{2}x^2 + C \Big|_0^4 \right)$$

$$= \pi (8 - 0) = \boxed{8\pi}$$

(2) $y = x^2 + 1,\ x = -1,\ x = 1,\ y = 0$

revolve around x-axis



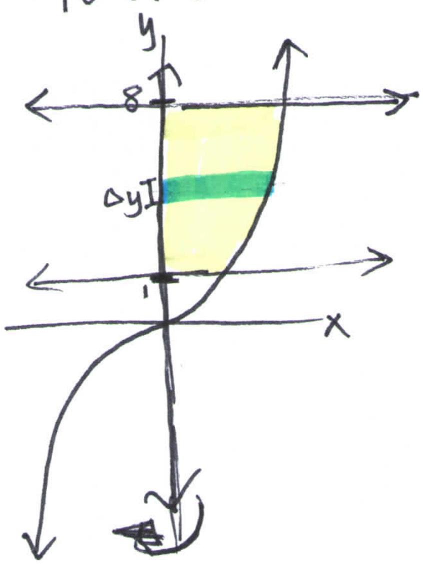
$$V = \pi \int_{-1}^{1} (x^2+1)^2 \, dx = \frac{56\pi}{15}$$

11.729

$y = x^3 \rightarrow x = \sqrt[3]{y}$

(3) $y = x^3,\ y = 1,\ y = 8,\ x = 0$

revolve around y-axis

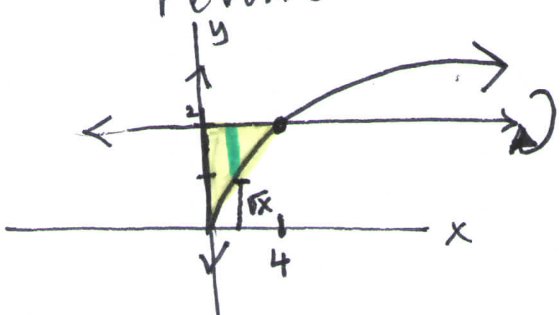


$$V = \pi \int_{1}^{8} \left(\sqrt[3]{y}\right)^2 dy = \frac{93}{5}\pi$$

58.434

(4) $y = \sqrt{x},\ y = 2,\ x = 0$

revolve around $y = 2$



$$V = \pi \int_{0}^{4} (2 - \sqrt{x})^2 \, dx = \frac{8}{3}\pi$$

$= 8.378$