

Notes -- Intro to Probability & The Counting Principle



Basic Probability

Things to Know ...



Tossing a 2-sided Coin

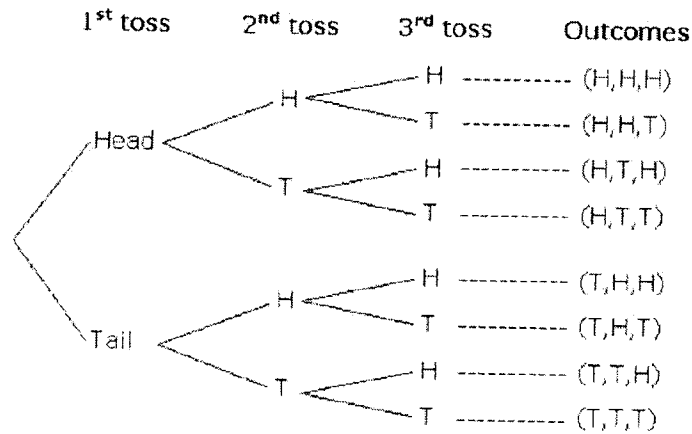
If you have multiple tosses, a tree diagram may help ...



Heads



Tails



Selecting a Card from a Standard Deck

Standard Deck of 52 Playing Cards:

Diamonds (Red): 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦ A♦

Hearts (Red): 2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥ A♥

Clubs (Black): 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣ A♣

Spades (Black): 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠ A♠

There are 52 cards in a standard deck with jokers removed.

There are 4 suits: Spades, Clubs, Hearts, Diamonds

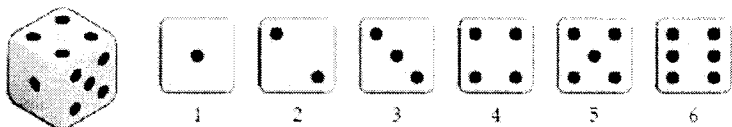
There are 26 black cards, and 26 red cards.

Each suit has 13 cards, each of a different rank.

Face cards are Jacks, Queens, and Kings.

There are 12 face cards in a deck.

Rolling a Standard Die



	1	2	3	4	5	6
1	1,1 ₂	1,2 ₃	1,3 ₄	1,4 ₅	1,5 ₆	1,6 ₇
2	2,1 ₃	2,2 ₄	2,3 ₅	2,4 ₆	2,5 ₇	2,6 ₈
3	3,1 ₄	3,2 ₅	3,3 ₆	3,4 ₇	3,5 ₈	3,6 ₉
4	4,1 ₅	4,2 ₆	4,3 ₇	4,4 ₈	4,5 ₉	4,6 ₁₀
5	5,1 ₆	5,2 ₇	5,3 ₈	5,4 ₉	5,5 ₁₀	5,6 ₁₁
6	6,1 ₇	6,2 ₈	6,3 ₉	6,4 ₁₀	6,5 ₁₁	6,6 ₁₂

Basic Probability

probability of an event = $\frac{\text{the \# of outcomes which make up the event}}{\text{the total \# of outcomes}}$

Example 1 Roll a die. Find:

A. p(even #) $\frac{3}{6} = \frac{1}{2}$

B. p(5) $\frac{1}{6}$

C. p(zero) $\frac{0}{6} = 0$

D. p(not 3) $\frac{5}{6}$

$\hookrightarrow 1 - p(3) = 1 - \frac{1}{6} = \frac{5}{6}$

Example 2 Toss a coin. Find:

A. p(head) $\frac{1}{2}$

B. p(tails) $\frac{1}{2}$

Example 3 Draw a card. Find:

A. p(heart) $\frac{13}{52} = \frac{1}{4}$

B. p(black) $\frac{26}{52} = \frac{1}{2}$

C. p(not diamond) $\frac{39}{52} = \frac{3}{4}$

D. p(red queen) $\frac{2}{52} = \frac{1}{26}$

E. p(face card) $\frac{12}{52} = \frac{3}{13}$

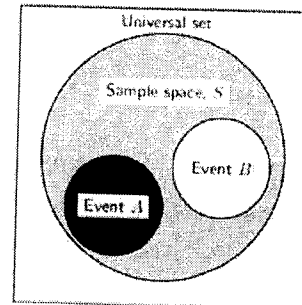
$1 - p(\text{diam.}) = 1 - \frac{13}{52} = \frac{3}{4}$

The Fundamental Counting Principle

If task A can be completed in "a" ways, and task B can be completed in "b" ways, then task A followed by task B can be completed in $a \cdot b$ ways.

outcome --the result of a single trial

Sample space --the set of all possible outcomes



Example 4 How many ways can a president and a secretary be chosen for a 4-member club? Assume the same person can't be both. Suppose the club members are Andy, Bill, Cathy, and Dawn.

P	S	P	S
A	B	C	A
A	C	C	B
A	D	C	D
B	A	D	A
B	C	D	B
B	D	D	C

sample space

12

$4 \cdot 3 = 12$

Example 5 How many outcomes are possible?

- You flip a coin. 2
- You flip a coin and roll a 6-sided die. $2 \cdot 6 = 12$
- You flip a coin, roll a die, and pick a card from a standard deck.

$$2 \cdot 6 \cdot 52 = 624$$

Example 6 At dinner you must choose an appetizer, an entrée, and a dessert. Your choices are:

appetizer—salad, fruit, or cheese

entrée—chicken or steak

dessert—pie or cake

How many outcomes are possible? $3 \cdot 2 \cdot 2 = 12$

Example 7 A license plate is composed of 2 letters followed by 3 numbers followed by 2 letters. How many outcomes are possible?

$$\underline{26} \cdot \underline{26} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{26} \cdot \underline{26}$$
$$4,569,760,000$$

Example 8 A license plate is composed of 2 letters followed by 3 numbers followed by 2 letters. You cannot repeat letters or numbers. How many outcomes are possible?

$$\underline{26} \underline{25} \underline{10} \underline{9} \underline{8} \underline{24} \underline{23}$$
$$258,336,000$$

Example 9 A combination is composed of 3 digits. If you can repeat digits, how many combinations are possible?

$$\underline{10} \cdot \underline{10} \cdot \underline{10} = 1000$$

Example 10 A combination is composed of 3 digits. If you cannot repeat digits, how many combinations are possible?

$$10 \cdot 9 \cdot 8 = 720$$