Notes -- *Geometric Distributions*

In the case of the binomial distribution, the number of trials was predetermined. Sometimes, however, we wish to know the number of trials needed before a certain outcome occurs. For example, we wish to play until we win or until we lose; you roll dice until you get an 11; a mechanic waits for the first plane to arrive at the airport that needs repair; a basketball player shoots until he makes it. These situations fall under the geometric distribution.

What are the four major principles that allow us to identify a geometric distribution?

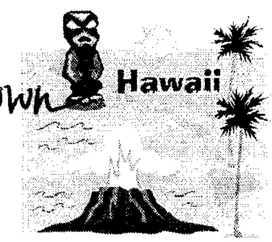
1. # trials is unknown
2. success / failure
3. p(success) is the same for each trial
4. trials are independent

If X has a geometric distribution with probability p of success and (1- p) of failure on each observation, the possible values of X are 1, 2, 3, ... If n is any one of these values, then the probability that the first success will occur on the nth trial is $P(X = n) = (1-p)^{n-1}p$ expected value = 1/p

<u>Example 1</u>: On the leeward side of the island of Oahu in the small village of Nanakuli, about 80% of the residents are of Hawaiian ancestry (The Honolulu Advertiser). Suppose you fly to Hawaii and visit Nanakuli. What is the P(first villager you meet is Hawaiian)? What is the P(you don't meet a Hawaiian until the second villager)? Etc?

Question: Why does this situation satisfy the geometric setting?

# of villagers we have to meet is unknown



Label everything you know!

p = .80 q= .20 x = varies

Let's start of by filling in the following probability distribution table!

In order to determine the EXACT number for each probability we use Geometric pdf in our calculator.

P(X=k)= geometpdf (.80, X)

Build an appropriate probability distribution chart to answer the following questions.

X	0	1	2	3	4	5	6	7	8
P(X)	0	.8	.16	.032	.0064	.00128	2.56 × 10^{-4}	5.12 × 10^{-5}	1.024 × 10^{-5}

Let X=# of villagers you must meet until you meet an actual Hawaiian

P(X=1) =

P(X=2) =

P(X=3) =

P(X=4) =

P(X=5) =

When looking for an exact probability we will use: P(X=x)= **geometpdf(p, x)**

When looking for probabilities that are cumulative we will use: **P (X ≤x) = geometcdf(p, x)**

Using your **chart** find the following:

a. What is the probability of meeting a Hawaiian **by** the 6th villager? geometcdf(.8 , 6)
   $p(x \le 6) = p(1)+p(2)+p(3)+p(4)+p(5)+p(6) = .99968$

b. What is the probability it will take **more than** 4 villages before meeting a Hawaiian?
   $p(x>4) = p(5)+p(6)+p(7)+p(8) = .002$ 1-geometcdf(.8,4) .0016

c. What is the probability of not meeting an Hawaiian in the first 7 villagers?
   $1-p(\text{meet w/in 1st 7}) = 1-.999936 = 6.4\times10^{-5}$ 1-geometcdf(.8,7) 1.28×10^{-5} .0000128 0
   .000064 0

d. What is the expected value?
   $\frac{1}{.8} = 1.25$

Now, try the following by building a chart and check your answer by using the calculator options!

Example 2: A computer testing program is designed to present questions to the user until a correct answer is given. Suppose that each question has five possible answers, and that the user is guessing.

p(success) = .2

X	0	1	2	3	4	5	6	7
P(X)	0	.2	.16	.128	.1024	.08192	.065536	.05243

a. What is the probability that the user will have to answer 5 questions in order to get one question correct?
   $p(x=5) = \text{geometpdf}(.2, 5) = .082$

b. What is the probability that the user will have to answer **more than 4** questions to get one correct?
   $p(x>4) = 1-\text{geometcdf}(.2, 4) = .4096$

c. What is the probability that the user will get a correct answer **by** the 8th question?
   $p(x \le 8) = \text{geometcdf}(.2, 8) = .832$

d. What is the expected value? $\frac{1}{.2} = 5$