

NOTES--Graph Theory Vocabulary/Euler Paths & Circuits

Important Terms to Know . . .



graph-- finite set of dots & connecting links

vertex-- dot on a graph

edge -- a link on a graph that must connect 2 different vertices

path -- a connected sequence of edges showing a route on a graph that starts at a vertex and ends at a vertex (described by naming in order the vertices visited in transversing it)

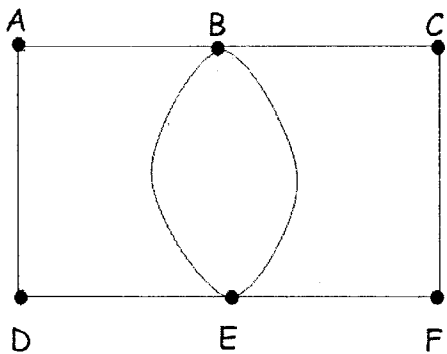
circuit-- a path that starts and ends at the same vertex

valence/degree-- the # of edges that meet at that vertex

connected-- there is a path from one vertex to any other vertex

loop-- counts as 1 edge & contributes 2 to the degree

Example 1 Complete the following information for the graph:



6 vertices 8 edges connected? yes

VERTEX	VALENCE
A	2
B	4
C	2
D	2
E	4
F	2

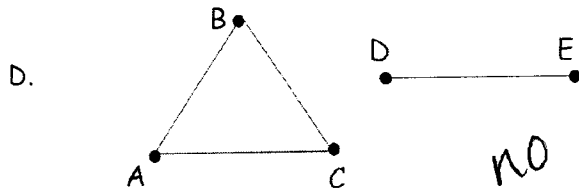
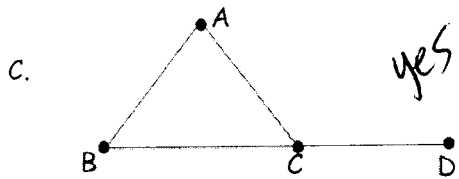
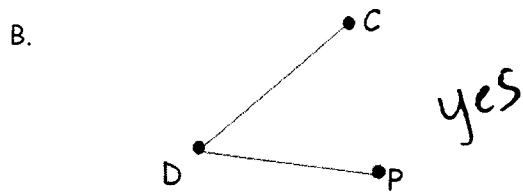
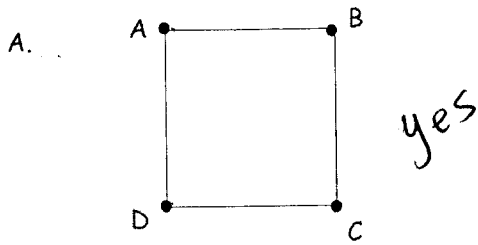
Example 2 Complete the following information for the graph:

2 vertices 2 edges connected? yes



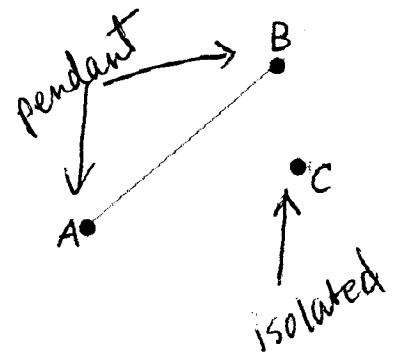
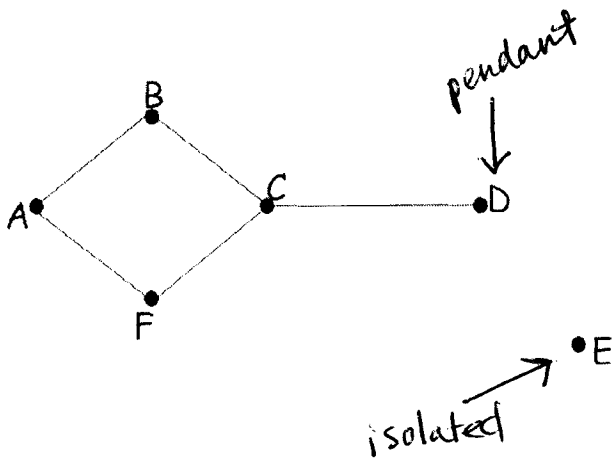
vertex	valence
A	1
B	3

Example 3 Determine if graph is connected or not:

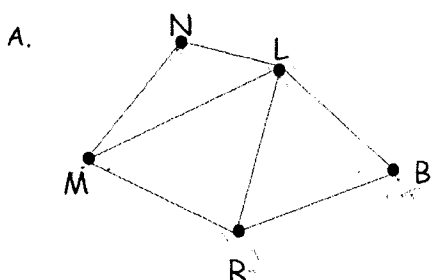


pendant vertex-- a vertex with degree 1.

isolated vertex-- a vertex with degree 0.



Example 4 Find the requested information about each graph:



of vertices = 5

of edges = 7

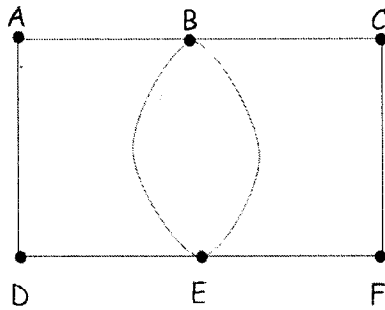
some paths from N to B: NLB, NLRB, NmRB, NMLNMRB

some circuits:

MLNM
MRBLNM

BRLB

B.



of vertices = 6

of edges = 8

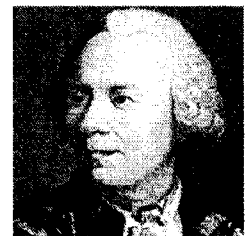
some paths from A that cover all of the edges:

ADEFCEBEBA

ABEBCFE DA

Euler path-- a path that travels through every edge once

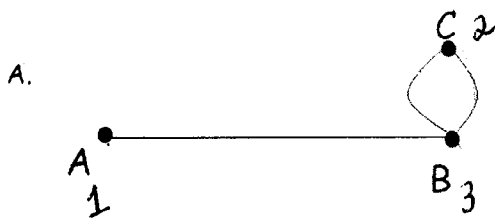
Euler circuit-- a circuit that travels through each edge of a graph only once



Things to know . . .

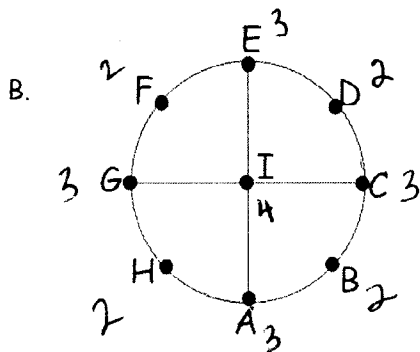
- If a graph has all even valences, then it has at least one Euler circuit. In other words, if a graph has an odd vertex, then it has no Euler circuit.
- If a graph has exactly 2 odd vertices, then it has at least one Euler path which starts at one vertex and ends at the other.

Example 5 If possible, name an Euler path and an Euler circuit for each graph:



Euler path: ABCB BCBA

Euler circuit: none



Euler path: none

Euler circuit: none



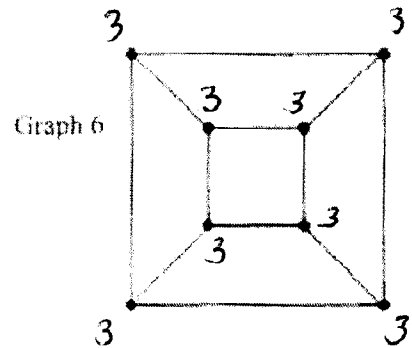
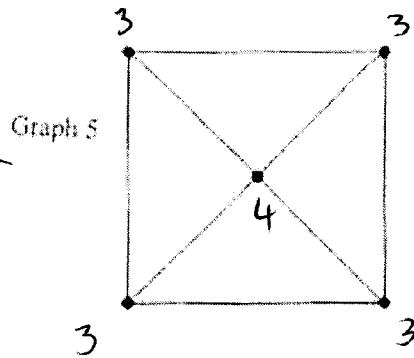
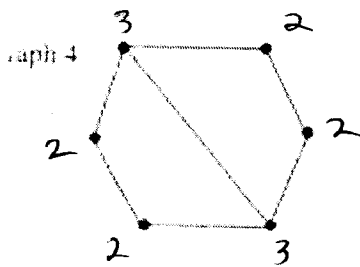
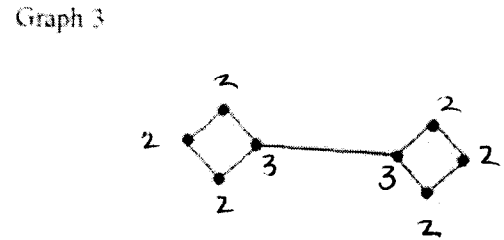
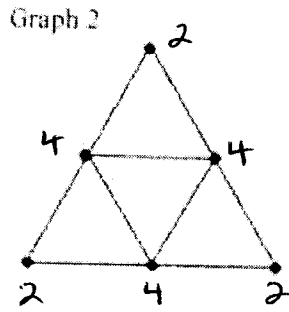
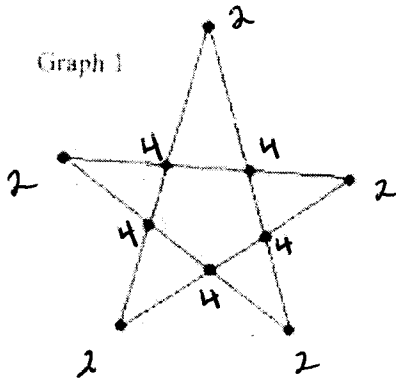
Euler path: ABB

Euler circuit: none

Euler Paths and Circuits

Question: What is the relationship between the nature of the vertices and the kind of path/circuit that the graph contains?

Directions: Complete the chart below using the given graphs. Use the chart to finish the statements.



Graph #	Number of vertices with an odd degree	Number of vertices with an even degree	Euler path, Euler circuit, or neither?
1	0	10	EC
2	0	6	EC
3	2	6	EP
4	2	4	EP
5	4	1	neither
6	8	0	neither

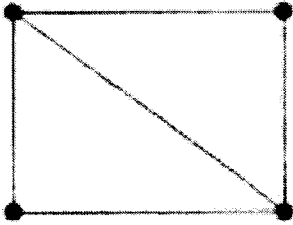
A graph with all even vertices has an Euler circuit.

A graph with 2 odd vertices and some even vertices has an Euler path.

A graph with more than 2 odd vertices has neither.

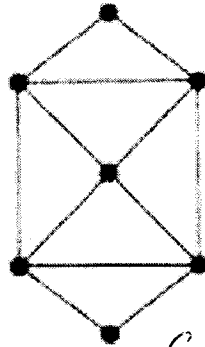
Determine which graphs have Euler paths, Euler circuits, or neither.

A.



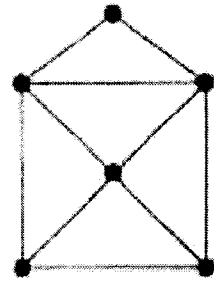
EP

B.



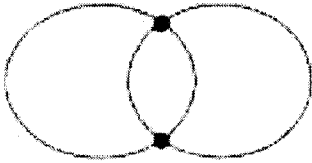
EC

C.



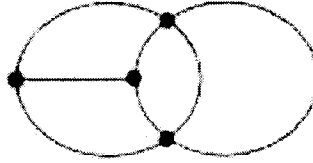
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D.



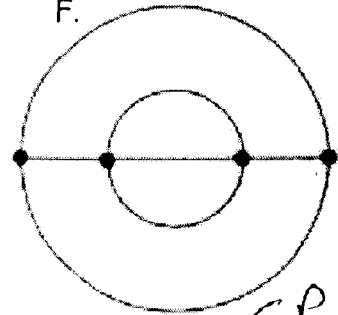
EC

E.



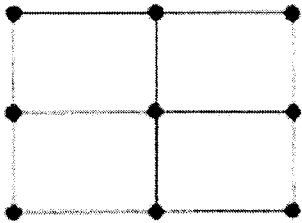
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F.



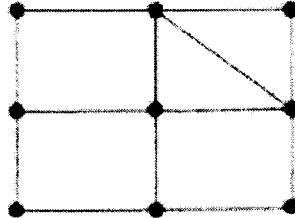
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G.



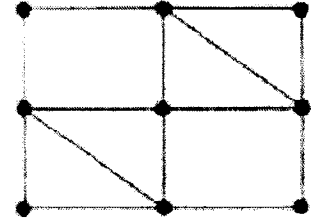
neither

H.



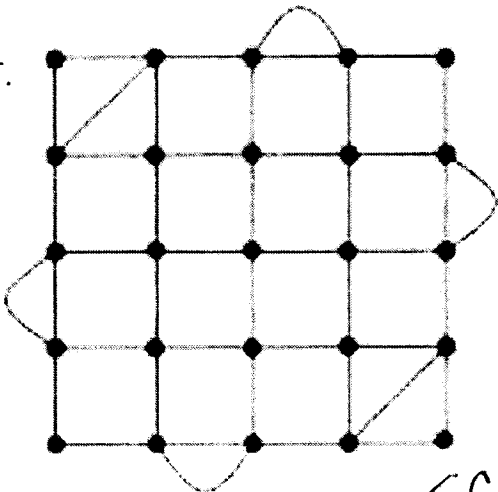
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I.



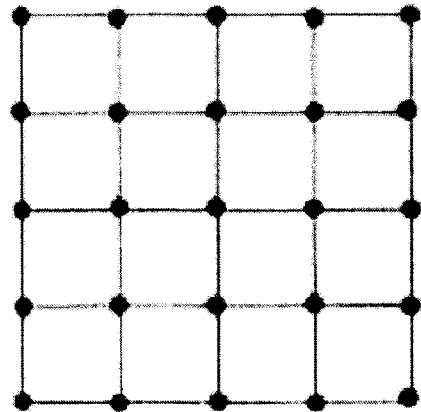
EC

J.



EC

K.



neither