

Plurality Method: the Candidate with the most first-place votes wins

Advantages:

- Easy to determine
- Easy to understand
- Only need 1st choice (not necessarily a ranked ballot)

Most political elections in the USA are determined using the plurality method.

Sometimes using plurality has problems . . .

How many first place votes did our winner get?

A 14 votes

Where is our winner on EVERYONE ELSE's BALLOT?

Number of voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Example 2 The band has the choice to perform at 5 different bowl games: the Rose Bowl (R), the Hula Bowl (H), the Fiesta Bowl (F), the Orange Bowl (O), and the Sugar Bowl (S).

Who is the PLURALITY winner?

Rose Bowl

Is there a better choice? Why?

Hula - more preferred by everyone

R 49
H 48
F 3
O 0
S 0

100 votes

Number of voters	49	48	3
1st choice	R	H	F
2nd choice	H	S	H
3rd choice	F	O	S
4th choice	O	F	O
5th choice	S	R	R

Example 3 Below is a preference schedule for Mr. Tucker's kindergarten class when they ranked their favorite colors among the choices Red, Blue and Purple.

1st	P	P	B	B	R	R
2nd	R	B	P	R	B	P
3rd	B	R	R	P	P	B
	9	4	14	10	16	4

Who is the plurality winner?

Blue

Is the plurality winner a majority winner?

$\frac{1}{2}(57) = 28.5$
majority: ≥ 29

P - 13
R - 20
B - 24

Plurality with Elimination (also called sequential run-off)

- carried out in rounds
- After each round of voting the candidate (or alternative) with the fewest first place votes is eliminated and a new round of voting is done with the remaining candidates
- When only two candidates remain in a round, the candidate with the most votes wins the election
- Note: At any time during this process if a candidate has a majority of first-place votes, then that candidate is the winner
- For an N candidate election, Plurality with Elimination requires N-1 rounds

Example 4 The mayor of Smallville is being chosen in a Plurality with Elimination election. The four candidates are Paul (the town barber), Rita (head of the town council), Sarah (Superintendent of Education), and Tim (former District Attorney).

500 registered voters cast their preference ballots. The results are summarized in the preference schedule below.

	# of Voters			
Place	130	120	100	150
1st	P	T	T	S
2nd	R	R	R	R
3rd	S	S	P	P
4th	T	P	S	T

- A. How many rounds will it take to determine a winner? **3**
- B. Who is the winner using Plurality with Elimination?

Round 1

P	130
R	0
S	150
T	220

eliminate R

Round 2

P	130
S	150
T	220

elim. P

Round 3

S	280
T	220

S wins

Example 5

Preference	12 voters	9 voters	3 voters	8 voters
1 st	Charles	Bonnie	Adam	Adam
2 nd	Adam	Charles	Charles	Bonnie
3 rd	Bonnie	Adam	Bonnie	Charles

- A. How many rounds will it take to determine a winner? **2**
- B. Who is the winner using Plurality with Elimination?

Round 1

A	11
B	9
C	12

eliminate B

Round 2

A	11
C	21

C wins

Example 6 Who is the Plurality with Elimination winner?

Preference	15 voters	7 voters	13 voters	5 voters	2 voters
1st	Helen	Eddie	Grover	Donna	Eddie
2nd	Donna	Donna	Flora	Grover	Flora
3rd	Eddie	Grover	Eddie	Flora	Grover
4th	Flora	Flora	Donna	Eddie	Donna
5th	Grover	Helen	Helen	Helen	Helen

Round 1

D	5
E	9
F	0
G	13
H	15

elim.
F

Round 2

D	5
E	9
G	13
H	15

elim.
D

Round 3

E	9
G	18
H	15

elim E

Round 4

G	27
H	15

G wins

Borda Count

- ❖ if there are m candidates, then for each vote, m points are assigned to the 1st choice, (m - 1) points are assigned to the 2nd choice, and so on
- ❖ the candidate that receives more points in total than any other is declared the winner
- ❖ invented in 1770 by the French mathematician and physicist Jean-Charles Borda

Advantages

- ❖ Incorporates all information from a preference ballot
- ❖ takes candidate with best average ranking
- ❖ preferable when comparing a large number of candidates



Example 7

The Math Appreciation Society is electing its president. The candidates are Alisha (A), Boris (B), Carmen (C), and Dave (D). Each of the 37 members votes with a preference ballot. Who should be the winner using a Borda Count?

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

$$A: 14(4) + 0(3) + 0(2) + 23(1) = 79$$

$$B: 4(4) + 24(3) + 9(2) + 0(1) = 106$$

$$C: 11(4) + 8(3) + 18(2) + 0(1) = 104$$

$$D: 8(4) + 5(3) + 10(2) + 14(1) = 81$$

B wins

Example 8

Determine the Borda count winner.

Preference	12 voters	9 voters	3 voters	8 voters
1 st	Charles	Bonnie	Charles	Adam
2 nd	Adam	Charles	Adam	Bonnie
3 rd	Bonnie	Adam	Bonnie	Charles

$$A: 8(3) + 15(2) + 9(1) = 63$$

$$B: 9(3) + 8(2) + 15(1) = 58$$

$$C: 15(3) + 9(2) + 8(1) = 71$$

Charles wins

Run-off Method

- ❖ Voters cast a preference ballot.
- ❖ Eliminate all candidates except the two (2) with the most 1st choice votes.
- ❖ Reassign votes from eliminated candidates by giving to the remaining candidate ranked higher.

Example 9

The Math Appreciation Society is electing its president. The candidates are Alisha (A), Boris (B), Carmen (C), and Dave (D). Each of the 37 members votes with a preference ballot. Who should be the winner using the Run-off method?

Number of Voters	14	10	8	4	1
1st choice	A	C	B	B	C
2nd choice	B	B	C	B	D
3rd choice	C	B	B	C	B
4th choice	B	A	A	A	A

A - 14
~~B - 4~~
 C - 11
~~D - 8~~ } elim
 A - 14
 C - 23
C wins

Example 10 Determine a winner using the Run-off method.

A 49
 B 48
 C 3
 D 0 elim.

A 49
 B 51
B wins

Number of voters	49	48	3
1st choice	A	A	B
2nd choice	A	B	D
3rd choice	A	B	D
4th choice	A	B	A

Example 11

Ms. Powell is rewarding her ICM students by giving them an ice cream party. She takes a poll of favorite ice cream flavors among her students where A = vanilla, B = chocolate, C = strawberry, and D = mint chocolate chip (her favorite). Who is the run-off method winner?

Number of voters	27	19	8	15	2
1st choice	B	A	D	D	A
2nd choice	B	B	D	A	D
3rd choice	A	D	A	D	D
4th choice	D	B	B	B	B

A 21
 B 27
 C 15
 D 8
 elim.
 A 44
 B 27
A wins
 5