limits → a

- plug in #
- $\frac{0}{0}$ algebra (factor, find common denom., * by conjugate)
- $\frac{\#\text{ not } 0}{0}$ DNE (direction?)

one-sided limits

$\lim_{x \to \#^-} f(x)$ from left

$\lim_{x \to \#^+} f(x)$ from right

continuity

$\lim_{x \to \#^+} f(x) = \lim_{x \to \#^-} f(x)$

$f(\#)$ exists

equal

limits → ∞ or −∞ (end behavior)

top heavy: $\infty, -\infty$, DNE

bottom heavy: 0

~~equal~~ degrees: ratio of leading coefficients

limit defn. of derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

derivative rules

constant
"x" rule
* power rule
constant multiple
sum/diff rule

Introduction to Calculus—test review

Evaluate the following limits. If the limit does not exist, give the direction (if it has one).

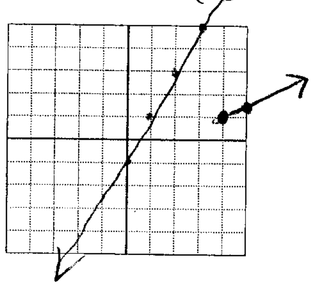
1. $\lim\limits_{x\to 0^-}\frac{1}{x}$ $-\infty$
2. $\lim\limits_{x\to 0^+}\frac{1}{x}$ ∞
3. $\lim\limits_{x\to 0}\frac{1}{x}$ DNE
4. $\lim\limits_{x\to \infty}\frac{1}{x}$ 0

5. $\lim\limits_{x\to 0}\frac{1}{x^2}$ ∞
6. $\lim\limits_{x\to \infty}\frac{2-6x}{5x+1}$ $-\frac{6}{5}$
7. $\lim\limits_{x\to -\infty}\frac{x}{x^3+2}$ 0
8. $\lim\limits_{x\to \infty}\frac{2x^2+1}{x}$ ∞

9. $\lim\limits_{x\to 0}\frac{6x-9}{x^3-12x+3}$ -3
10. $\lim\limits_{x\to 6}\frac{x+6}{x^2-36}$ DNE
11. $\lim\limits_{x\to -2}\frac{x^2-4x+4}{x^2+x-6}$ -4

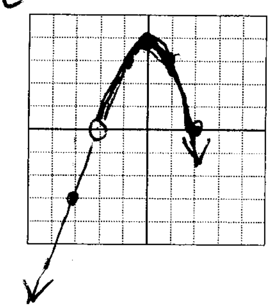
12. $\lim\limits_{x\to \infty} 3$ 3
13. $\lim\limits_{x\to 9}\frac{9-x}{3-\sqrt{x}}$ 6
14. $\lim\limits_{x\to -3}\frac{x^2-9}{x+3}$ -6

15. $\lim\limits_{x\to 4} f(x)$, $f(x)=\begin{cases}\frac{1}{2}x-1, & x\ge 4\\ 2x-1, & x<4\end{cases}$
DNE



jump discont. at $x=4$

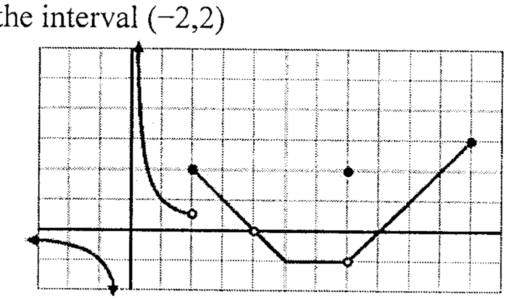
16. $\lim\limits_{x\to -2} f(x)$, $f(x)=\begin{cases}-x^2+4, & x>-2\\ 3x+6, & x<-2\end{cases}$ hole at $x=-2$
0



Refer to the graph to evaluate the following:

1. ∞ $\lim\limits_{x\to 0^+} f(x)$
2. $-\infty$ $\lim\limits_{x\to 0^-} f(x)$
3. DNE $\lim\limits_{x\to 0} f(x)$
4. 2 $\lim\limits_{x\to 2^+} f(x)$
5. $1/2$ $\lim\limits_{x\to 2^-} f(x)$
6. DNE $\lim\limits_{x\to 2} f(x)$
7. -1 $\lim\limits_{x\to 7^+} f(x)$
8. -1 $\lim\limits_{x\to 7^-} f(x)$
9. -1 $\lim\limits_{x\to 7} f(x)$
10. DNE $\lim\limits_{x\to 11^+} f(x)$
11. 3 $\lim\limits_{x\to 11^-} f(x)$
12. DNE $\lim\limits_{x\to 11} f(x)$
13. 0 $\lim\limits_{x\to 4} f(x)$
14. -1 $\lim\limits_{x\to 5} f(x)$
15. DNE $f(0)$
16. 2 $f(2)$
17. DNE $f(4)$
18. 2 $f(7)$
19. true True or False: $\lim\limits_{x\to c} f(x)$ exists for every c in the interval (2,10)
20. false True or False: $\lim\limits_{x\to c} f(x)$ exists for every c in the interval (−2,2)
21. List each x value where a discontinuity occurs and describe the type of discontinuity.

$x=0$ infinite
$x=2$ jump
$x=4$
$x=7$ } hole (removeable)



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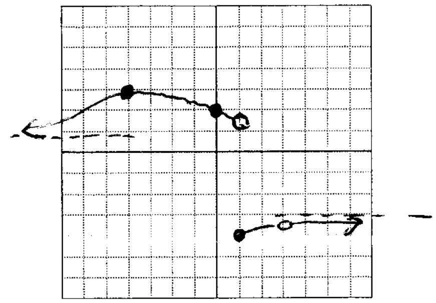
Draw a graph with the following conditions:

- f(0) = 2
- f(1) = -4
- f(-4) = 3
- at f(1) there is a non-removable discontinuity (jump)
- at f(3) there is a removable discontinuity (hole)
- $\lim_{x \to -\infty} f(x) = 1$
- $\lim_{x \to \infty} f(x) = -3$



Use the limit definition to find each derivative:

a. f(x) = 3x² – 5x + 1

$$f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 - 5(x+h) + 1 - (3x^2 - 5x + 1)}{h} = \lim_{h \to 0} \frac{3x^2 + 6hx + 3h^2 - 5x - 5h + 1 - 3x^2 + 5x - 1}{h}$$

$$= \lim_{h \to 0} \frac{6hx + 3h^2 - 5h}{h} = \lim_{h \to 0} 6x + 3h - 5 = \boxed{6x - 5}$$

b. $g(x) = \frac{-3}{x+2}$

$$g'(x) = \lim_{h \to 0} \frac{\frac{-3}{x+h+2} - \frac{-3}{x+2}}{h} = \lim_{h \to 0} \frac{\frac{-3(x+2) + 3(x+h+2)}{(x+h+2)(x+2)}}{h}$$

$$= \lim_{h \to 0} \frac{-3x - 6 + 3x + 3h + 6}{(x+h+2)(x+2)} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{3h}{(x+h+2)(x+2)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{3}{(x+h+2)(x+2)} = \boxed{\frac{3}{(x+2)^2}}$$

c. $p(x) = \sqrt{3x+2}$

$$p'(x) = \lim_{h \to 0} \frac{\sqrt{3(x+h)+2} - \sqrt{3x+2}}{h} = \lim_{h \to 0} \frac{\left(\sqrt{3(x+h)+2} - \sqrt{3x+2}\right)\left(\sqrt{3(x+h)+2} + \sqrt{3x+2}\right)}{h\left(\sqrt{3(x+h)+2} + \sqrt{3x+2}\right)}$$

$$= \lim_{h \to 0} \frac{3x + 3h + 2 - (3x+2)}{h\left(\sqrt{3(x+h)+2} + \sqrt{3x+2}\right)} = \lim_{h \to 0} \frac{3}{\sqrt{3(x+h)+2} + \sqrt{3x+2}} = \boxed{\frac{3}{2\sqrt{3x+2}}}$$

Find the derivative of each function:

a. $y = 9$

$y' = 0$

b. $y = 5x^3 - 4x^2 + 3x - 11$

$y' = 15x^2 - 8x + 3$

c. $f(x) = \frac{1}{x} + \frac{3}{x^2}$

$f'(x) = -\frac{1}{x^2} - \frac{6}{x^3}$

d. $g(x) = (3x^2 + 4)^2 = (9x^4 + 24x^2 + 16)$

$g'(x) = 36x^3 + 48x$

~~e.~~ $y = \frac{3x^2 - 2x + 7}{5x^3 - 2x}$

~~f.~~ $f(x) = (4x^2 - 8x + 9)(5x^4 - 6x^3 + 7x^2 - 10x + 21)$

g. $y = 7\sqrt[3]{x^2} + \frac{2}{3}\sqrt{x} = 7x^{2/3} + \frac{2}{3}x^{1/2}$

$y' = \frac{14}{3}x^{-1/3} + \frac{1}{3}x^{-1/2} = \frac{14}{3\sqrt[3]{x}} + \frac{1}{3\sqrt{x}}$

~~h.~~ $y = \frac{2x^4}{x-9}$

i. $f(x) = 3x^2(7x^3 - 8x + 3) = 21x^5 - 24x^3 + 9x^2$

$f'(x) = 105x^4 - 72x^2 + 18x$

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