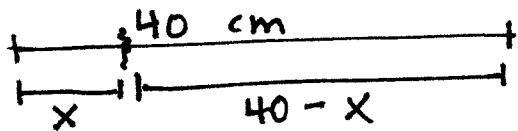


Optimization Day 2

EX 1 piece of wire 40cm long
 cut into 2 pieces to form a circle & a square
 Find the lengths of each piece that cause
 the sum of the area of the circle &
 the area of the square to be a minimum.



use x for \odot , use $40-x$ for \square

$$\begin{aligned} C &= x \\ 2\pi r &= x \\ r &= \frac{x}{2\pi} \end{aligned}$$

$$\begin{aligned} P &= 40 - x \\ S &= \frac{40 - x}{4} \end{aligned}$$

$$\begin{aligned} A &= \pi r^2 + S^2 = \pi \left(\frac{x}{2\pi}\right)^2 + \left(\frac{40-x}{4}\right)^2 \\ &= \pi \cdot \frac{x^2}{4\pi^2} + \left(10 - \frac{1}{4}x\right)^2 \end{aligned}$$

$$A = \underbrace{\frac{1}{4\pi} \cdot x^2}_{\text{circle}} + 100 - 5x + \underbrace{\frac{1}{16}x^2}_{\text{square}}$$

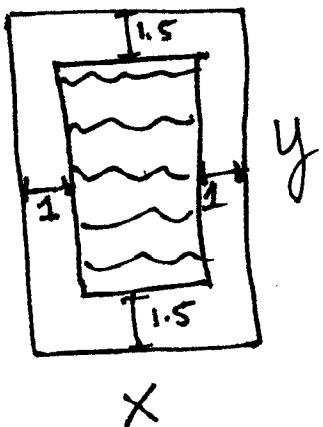
$$A' = \underbrace{\frac{1}{4\pi} \cdot 2x}_{\frac{1}{2\pi}} - 5 + \frac{1}{8}x = 0$$

$$\begin{aligned} \frac{1}{2\pi}x + \frac{1}{8}x &= 5 & - & + \\ x \left(\frac{1}{2\pi} + \frac{1}{8}\right) &= 5 & \frac{40\pi}{4+\pi} & \leftarrow \min \\ \text{circle: } \frac{40\pi}{4+\pi} \text{ cm} & & 17.596 & \\ \text{sq: } \left(40 - \frac{40\pi}{4+\pi}\right) \text{ cm} & & X = \frac{5}{\frac{1}{2\pi} + \frac{1}{8}} & = \frac{5}{\frac{4+\pi}{8\pi}} = \frac{40\pi}{4+\pi} \end{aligned}$$

EX2

rectangular page is to have 24in^2 of print.
 top & bottom margins = 1.5 in each
 side margins = 1 in each

Find the dimensions of the page so
 that the least amount of paper is used.



$$A_{\text{page}} = xy \text{ minimize}$$

$$A_{\text{print}} = (x-2)(y-3) = 24$$

$$y-3 = \frac{24}{x-2}$$

$$y = 3 + \frac{24}{x-2}$$

$$A = x \left(3 + \frac{24}{x-2} \right) = x \left(\frac{3(x-2)+24}{x-2} \right) = x \left(\frac{3x-6+24}{x-2} \right)$$

$$A = \frac{3x^2 + 18x}{x-2}$$

$$A' = \frac{(x-2)(6x+18) - (3x^2 + 18x)(1)}{(x-2)^2}$$

$$3x^2 - 12x - 36 = 0 \quad = \quad \frac{6x^2 + 6x - 36 - 3x^2 - 18x}{(x-2)^2}$$

$$3(x^2 - 4x - 12) = 0$$

$$3(x-6)(x+2) = 0 \quad = \quad \frac{3x^2 - 12x - 36}{(x-2)^2} \quad \text{undef } x=2$$

$$x = 6, -2$$

$$\frac{-1}{-2} = \frac{1}{2} + \frac{1}{6} \leftarrow \min \quad y = 3 + \frac{24}{6-2} = 9 \quad \boxed{6\text{in by } 9\text{in}}$$