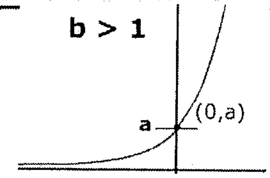
NOTES--APPLICATIONS OF EXPONENTIAL AND LOGARITHMIC EQUATIONS

Simple Growth/Decay Model

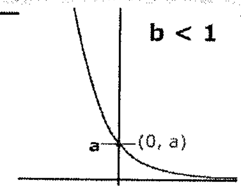
$f(x) = a \cdot b^x$

a & b are positive
$b > 1 \rightarrow$ growth
$b < 1 \rightarrow$ decay

Exponential Growth



Exponential Decay



The Population Growth/Decay Model
(use when the population changes at a constant rate)

$p(t) = p_0(1 + r)^t$

p_0 = initial population   r = rate of change as a decimal   t = time

Half-Life Model

"half-life" is the time it takes for a sample to be half of the amount it was before

$A(t) = A_0 \cdot e^{-kt}$

A_0 = initial amount   k is a constant   t = time

OR

$A(t) = A_0 \cdot (0.5)^{t/n}$

A_0 = initial amount   t = time   n = the half life

Newton's Law of Cooling

$T(t) = T_m + (T_0 - T_m)e^{-kt}$

T_m = temperature of the medium T_0 = initial temperature of the object
t = time   k is a constant

Chemical Acidity

$pH = -\log[H^+]$

H^+ = hydrogen-ion concentration

If pH < 7 → acid
If pH = 7 → neutral
If pH > 7 → base

Sound Intensity Model

$dB = 10\log\left(\frac{I}{1 \times 10^{-12}}\right)$

I = intensity (in watts per square meter)

Finance Models

"compounded annually" $A = P(1 + r)^t$
"compounded in periods" $A = P(1 + \frac{r}{k})^{tk}$
"compounded continuously" $A = Pe^{rt}$

P = principal amount   r = rate as a decimal   t = time (years)
k = # times compounded per year

Example 1 Suppose that the half-life of a compound is 20 days with 5 grams present initially. Find the time when there will be 1 gram remaining.

$A(t) = A_0 (0.5)^{t/h}$ $\quad 1 = 5(0.5)^{t/20}$ $\quad t = 46.44$ days

Example 2 Suppose you are given \$500 to invest. What annual interest rate compounded quarterly is required to double the money in 10 years?

$A = p\left(1+\frac{r}{k}\right)^{tk}$ $\quad 1000 = 500\left(1+\frac{r}{4}\right)^{10(4)}$ $\quad r = .0699$ $\quad r = 7\%$

Example 3 Given the data in the chart below, predict the panda population in ~~2020 for the Powell Zoo.~~ use 120

	year	population
0	1900	76.2
10	1910	92.2
20	1920	106.0
30	1930	123.2
40	1940	132.2
50	1950	151.3
60	1960	179.3
70	1970	203.3
80	1980	226.5
90	1990	248.7

expreg

$y = a * b^x$

$y = 80.075 \cdot 1.013^x$

$x = 120, \quad y = 381.186$

381 pandas

Example 4 Suppose a population in 1910 was 4200 and increased at 2.25% (.0225) per year. Estimate the population in 1930 ($t=20$) & 1945 ($t=35$). Predict when the population reached 20,000.

$P(t) = P_0 (1+r)^t$

$p(20) = 4200(1+.0225)^{20} = 6554$ people

$p(35) = 4200(1+.0225)^{35} = 9150$ people

$20000 = 4200(1+.0225)^t$

$t = 70.14$

year ~1980

Example 5 The noise level in a cafeteria is 125 dB. Find I.

$dB = 10 \log\left(\frac{I}{1\times10^{-12}}\right)$ $\quad 125 = 10 \log\left(\frac{I}{1\times10^{-12}}\right)$

$I = 3.162$ watts/m^2

Example 6 A hard-boiled egg at 96°C is placed in 16°C water to cool. Four minutes later the temperature of the egg is 45°C. Determine when the egg will be 20°C.

$T(t) = T_m + (T_0 - T_m)e^{-kt}$

$45^\circ = 16^\circ + (96^\circ - 16^\circ)e^{-k(4)}$

$k = .2536827$

$20^\circ = 16^\circ + 80^\circ e^{-.2536827t}$

$t = 11.81$ mins

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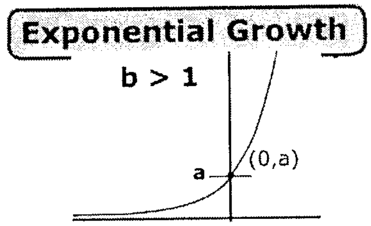

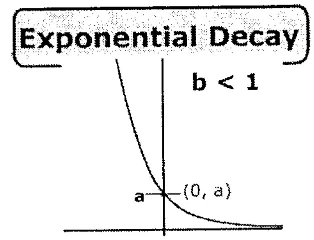

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