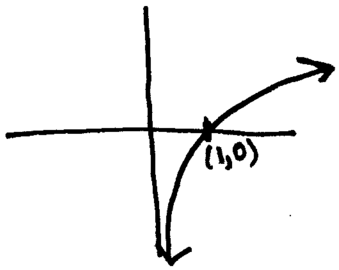
Natural Log Function

$y = \ln x$



D: $(0, \infty)$
R: $(-\infty, \infty)$
always increasing (strictly monotonic)
cont. on its domain
concave down

Properties of Logs

1. $\ln 1 = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
4. $\ln a^n = n \cdot \ln a$
5. $\ln e = 1$

Derivative Rules

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u' = \frac{u'}{u}$$

$$\frac{d}{dx}[\ln |u|] = \frac{u'}{u}$$

EX1 Find the derivative.

(A) $f(x) = \ln\left(\frac{x}{x+1}\right)$

$$f'(x) = \frac{1}{\frac{x}{x+1}} \cdot \frac{(x+1)(1)-(x)(1)}{(x+1)^2}$$

$$= \frac{x+1}{x} \cdot \frac{1}{(x+1)^2}$$

$$= \frac{1}{x(x+1)}$$

$f(x) = \ln x - \ln(x+1)$

$$f'(x) = \frac{1}{x} - \frac{1}{x+1}$$

$$= \frac{x+1-x}{x(x+1)}$$

$$= \frac{1}{x(x+1)}$$

(B) $y = \ln\sqrt{\frac{x-1}{x+1}} = \ln\left(\frac{x-1}{x+1}\right)^{1/2} = \frac{1}{2}\ln\left(\frac{x-1}{x+1}\right)$

$$= \frac{1}{2}\left[\ln(x-1) - \ln(x+1)\right]$$

$$y' = \frac{1}{2}\left[\frac{1}{x-1} - \frac{1}{x+1}\right] = \frac{1}{2}\left[\frac{x+1-(x-1)}{(x-1)(x+1)}\right] = \frac{1}{2}\left[\frac{2}{(x-1)(x+1)}\right]$$

$$= \frac{1}{x^2-1}$$

(C) $y = \ln|\sec x|$

$$y' = \frac{1}{\sec x} \cdot \sec x \tan x = \tan x$$

Logarithmic Differentiation

– use when you have a function raised to a function

OR

a complicated rational expression

1. set $f(x)$ equal to y and take $\ln$ of both sides
2. use log properties to expand the right side
3. take the derivative w.r.t. x on both sides
4. Solve for $\frac{dy}{dx}$
5. replace y with the original eqn.

EX2 (A) $f(x) = x^{\sin x}$ Find $f'(x)$.

$$\ln y = \ln x^{\sin x}$$

$$\ln y = (\sin x)(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = (\sin x)\left(\frac{1}{x}\right) + (\ln x)(\cos x)$$

$$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + (\ln x)(\cos x)\right]$$

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + (\ln x)(\cos x)\right]$$

(B) Find y' if $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$

$$\ln y = \ln\left(\frac{(x+1)(x+2)}{(x-1)(x-2)}\right) = \ln[(x+1)(x+2)] - \ln[(x-1)(x-2)]$$

$$\ln y = \ln(x+1) + \ln(x+2) - \ln(x-1) - \ln(x-2)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2}$$

$$\frac{dy}{dx} = y \cdot [\nearrow]$$

$$\frac{dy}{dx} = \frac{(x+1)(x+2)}{(x-1)(x-2}\left[\frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2}\right]$$

(C) $g(x) = \dfrac{(x^2+3)^3(4x-3)}{\sqrt{x^4+6}}$

$$\ln y = \ln\left[\frac{(x^2+3)^3(4x-3)}{\sqrt{x^4+6}}\right]$$

$$\ln y = 3\ln(x^2+3) + \ln(4x-3) - \tfrac{1}{2}\ln(x^4+6)$$

$$\frac{1}{y}\frac{dy}{dx} = 3 \cdot \frac{1}{x^2+3} \cdot 2x + \frac{1}{4x-3} \cdot 4 - \frac{1}{2} \cdot \frac{1}{x^4+6} \cdot 4x^3$$

$$\frac{dy}{dx} = \frac{(x^2+3)^3(4x-3)}{\sqrt{x^4+6}}\left[\frac{6x}{x^2+3} + \frac{4}{4x-3} - \frac{2x^3}{x^4+6}\right]$$