# Implicit Differentiation

eqn is not solved for $y$

Steps: 1) Differentiate both sides w.r.t. $x$

2) Put terms with $\frac{dy}{dx}$ on one side, everything else on the other side

3) Factor out $\frac{dy}{dx}$

4) Solve for $\frac{dy}{dx}$

**Ex 1** Find the derivative.

A. $12x^2 + y^2 = 144$

$$24x\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -24x$$

$$\frac{dy}{dx} = \frac{-24x}{2y} = -\frac{12x}{y}$$

B. $x^2y + y^2x = -2$

$$x^2 \cdot 1\frac{dy}{dx} + y \cdot 2x + y^2 \cdot 1 + x \cdot 2y\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = -2xy - y^2$$

$$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$$

$$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$$

C. $2x^3y^3 - y = x^{2/3}$

$2x^3 \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 6x^2 - 1\frac{dy}{dx} = \frac{2}{3}x^{-1/3}$

$6x^3y^2 \frac{dy}{dx} - \frac{dy}{dx} = \frac{2}{3\sqrt[3]{x}} - 6x^2y^3$

$\frac{dy}{dx}(6x^3y^2 - 1) = \frac{2}{3\sqrt[3]{x}} - 6x^2y^3$

$$\frac{dy}{dx} = \frac{\frac{2}{3\sqrt[3]{x}} - 6x^2y^3}{6x^3y^2 - 1}$$

D. $y = \sin(xy)$

$1 \cdot \frac{dy}{dx} = \cos(xy)\left[x \cdot \frac{dy}{dx} + y \cdot 1\right]$

$\frac{dy}{dx} = x\cos(xy) \cdot \frac{dy}{dx} + y\cos(xy)$

$\frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$

$\frac{dy}{dx}(1 - x\cos(xy)) = y\cos(xy)$

$$\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$$

EX 2 $3x + xy^3 = 4$

Find the slope of the curve at the point $(1,1)$.

deriv: $3 + x \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 1 = 0$

$$3xy^2 \frac{dy}{dx} = -3 - y^3$$

$$\frac{dy}{dx} = \frac{-3-y^3}{3xy^2}$$ general slope

at $(1,1)$: $\frac{-3-(1)^3}{3(1)(1)^2} = \boxed{\frac{-4}{3}}$

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$$xy^3 = 4 - 3x$$

$$y^3 = \frac{4-3x}{x}$$

$$y = \sqrt[3]{\frac{4-3x}{x}}$$