AP Calculus BC Review Notes Important Theorems

You should be able to . . .

- Apply theorems such as the Intermediate Value Theorem, Mean Value Theorem, and Rolle's Theorem

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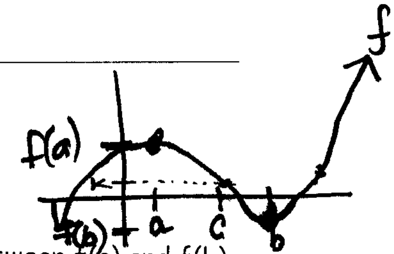
IVT – Intermediate Value Theorem



y-value

What it says: If f is continuous on the closed interval [a, b] and k is a number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c) = k.

What it means: If f is continuous between two points, and f(a) = j and f(b) = k, then for any c between a and b, f(c) will take on a value between j and k.

When to use it: Use to prove that a particular intermediate y value when you know two other y values on a continuous function.

x-intercepts $\Rightarrow y=0$

Ex 1) Show that $p(x) = 2x^3 - 5x^2 - 10x + 5$ has a root in the interval $[-1, 2]$.

p(x) is cont. over $[-1,2]$ -- polynom. are cont.

$p(-1) = -2-5+10+5 = 8$

$p(2) = 16-20-20+5 = -19$

$-19 < 0 < 8 \Rightarrow$ there's an x-value in $[-1,2]$ where $f(c)=0$ by the IVT

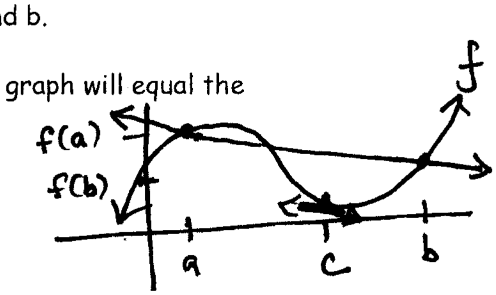
MVT – Mean Value Theorem

What it says: If f is continuous on the closed interval [a, b] and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$. ← difference quotient

slope of tangent = slope of secant

What it means: Given two points a and b, the slope between those points will be attained as an instantaneous slope (i.e. a derivative) by some point c that is between a and b.

When to use it: To prove that the slope between to distinct points on the graph will equal the derivative of the function at some point x between a and b.



Ex 2) Apply the MVT: $f(x) = x - \cos(x)$ over the interval $[0, 2\pi]$.

cont on $[0,2\pi]$? ✓
diff. on $(0,2\pi)$? ✓
} MVT applies

$f'(x) = 1+\sin x$

$f(0) = 0-\cos(0) = -1$

$f(2\pi) = 2\pi - \cos(2\pi) = 2\pi - 1$

$1+\sin x = \dfrac{2\pi-1-(-1)}{2\pi-0} = 1$

$\sin x = 0$

$x = \sin^{-1}(0)$

$x = 0, \pi$

$\boxed{\pi}$

13

Ex 3) Apply the MVT: $f(x) = x^3 - x^2$ over the interval $[-4, 4]$.

f cont on [-4,4] ? ✓
f diff. on (-4,4)? ✓ } MVT applies

$f'(x) = 3x^2 - 2x$

$f(-4) = -64 - 16 = -80$

$f(4) = 64 - 16 = 48$

$3x^2 - 2x = \frac{48 - {}^-80}{4 - {}^-4} = 16$

$3x^2 - 2x - 16 = 0$

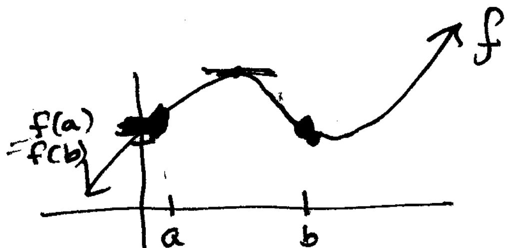
$(3x - 8)(x + 2) = 0$

$\boxed{x = 8/3 \quad x = -2}$

Ex 4) Explain why the MVT does not apply to $f(x) = x^{\frac{2}{3}}$ over the interval $[-8, 1]$.

not diff on (-8,1) at x=0 $\quad f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

MVT doesn't apply



zero

Rolle's Theorem

What it says: Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If f(a) = f(b), then there is at least one number c in (a, b) such that f'(c) = 0.

What it means: If a function has two places, a and b, where the y values are the same, then there will be a horizontal tangent somewhere between a and b.

When to use it: Use it the same way as the MVT. You could also apply it to prove a theoretical max or min between two x values if you can't actually see the graph.

Ex 5) Apply Rolle's Theorem to $f(x) = \sin x$ over the interval $[0, 2\pi]$.

cont on $[0, 2\pi]$? ✓
diff on $(0, 2\pi)$? ✓
$f(0) \stackrel{?}{=} f(2\pi)$ ✓
$\sin 0 \quad \sin(2\pi)$
$0 \quad 0$
} Rolle's thm applies

$f'(x) = \cos x = 0$

$x = \cos^{-1}(0)$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$