Integration By Parts

- use if you have a product of 2 functions of x
- rule: $\int u\,dv = uv - \int v\,du$
- hints: (1) whatever dv is, you need to be able to find v

  (2) it helps if du is simpler than u & if v is simpler than dv (or no more complicated)

- deriving the rule...

Let u & v be functions of x

product : uv

$$dx\left(\frac{d}{dx}[uv]\right) = \left(u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}\right)dx$$

$$\int d[uv] = \int (u\,dv + v\,du)$$

$$uv = \int u\,dv + \int v\,du$$

$$uv - \int v\,du = \int u\,dv$$

Logarithmic
Inverse trig
Algebraic
Trigonometric
Exponential

} pick for u

__EX1__ $\int xe^x\,dx$

$u = x \qquad dv = e^x\,dx$

$\frac{du}{dx} = 1 \qquad v = \int dv = \int e^x\,dx = e^x$

$du = dx$

$\int u\,dv = uv - \int v\,du$

$\int xe^x\,dx = x \cdot e^x - \int e^x\,dx$

$= \boxed{xe^x - e^x + C}$

$e^x(x-1) + C$

__EX2__ $\int \ln x\,dx$

$u = \ln x \qquad dv = dx$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \int 1\,dx = x$

$du = \frac{1}{x}\,dx$

$\int \ln x\,dx = (\ln x)(x) - \int \underbrace{x \cdot \frac{1}{x}}_{1}\,dx$

$= \boxed{x\ln x - x + C}$

EX3 $\int x^2 \ln x \, dx$

$u = \ln x \quad dv = x^2 dx$

$\frac{du}{dx} = \frac{1}{x} \quad v = \int x^2 dx = \frac{1}{3}x^3$

$du = \frac{1}{x} dx$

$$\int x^2 \ln x dx = \frac{1}{3}x^3 \cdot \ln x - \int \underbrace{\frac{1}{3}x^3 \cdot \frac{1}{x}}_{\frac{1}{3}x^2} dx$$

$$= \boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$$

EX4 $\int x^2 \sin x \, dx$

$u = x^2 \quad dv = \sin x \, dx$

$\frac{du}{dx} = 2x \quad v = \int \sin x dx = -\cos x$

$du = 2x dx$

$$\int x^2 \sin x dx = -x^2 \cos x - \int -2x \cos x \, dx$$

$$= -x^2 \cos x + 2 \int x \cos x \, dx$$

$u = x \quad dv = \cos x \, dx$

$\frac{du}{dx} = 1 \quad v = \int \cos x dx = \sin x$

$du = dx$

$x \sin x - \int \sin x \, dx$

$x \sin x - (-\cos x)$

$$= -x^2 \cos x + 2(x \sin x + \cos x) + C$$

EX 5

$$\int e^{2x}\cos 3x\,dx$$

$$u = \cos 3x \qquad dv = e^{2x}\,dx$$

$$\frac{du}{dx} = -3\sin 3x \qquad v = \int e^{2x}\,dx = \frac{1}{2}e^{2x}$$

$$du = -3\sin 3x\,dx$$

$$\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}\int e^{2x}\sin 3x\,dx$$

$$u = \sin 3x \qquad dv = e^{2x}\,dx$$

$$\frac{du}{dx} = 3\cos 3x \qquad v = \int e^{2x}\,dx = \frac{1}{2}e^{2x}$$

$$du = 3\cos 3x\,dx$$

$$\frac{1}{2}\sin 3x \cdot e^{2x} - \frac{3}{2}\int e^{2x}\cos 3x\,dx$$

algebra

$$\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}\left(\frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\int e^{2x}\cos 3x\,dx\right)$$

$$\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x - \frac{9}{4}\int e^{2x}\cos 3x\,dx$$

$$\frac{13}{4}\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x$$

$$\int e^{2x}\cos 3x\,dx = \boxed{\frac{4}{13}\left(\frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x\right) + C}$$