

Integration Review Day 2

Examples

$$\textcircled{1} \int \frac{t^2 - 3}{-t^3 + 9t + 1} dt$$

$$u = -t^3 + 9t + 1$$

$$\frac{du}{dt} = -3t^2 + 9 = -3(t^2 - 3)$$

$$-\frac{1}{3} du = (t^2 - 3) dt$$

$$-\frac{1}{3} \int \frac{du}{u} = -\frac{1}{3} \ln|u| + C$$

$$-\frac{1}{3} \ln|-t^3 + 9t + 1| + C$$

$$\textcircled{2} \int 3 \sin^2 x \cos x dx$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x dx$$

$$3 \int u^2 du$$

$$u^3 + C$$

$$(\sin x)^3 + C$$

$$\sin^3 x + C$$

$$\textcircled{3} \int x \sqrt{2x-1} dx$$

$$u = 2x-1 \rightarrow u+1 = 2x \quad x = \frac{u+1}{2}$$

$$\frac{du}{dx} = 2 \quad \frac{1}{2} du = dx$$

$$\frac{1}{2} \int \frac{u+1}{2} \sqrt{u} du$$

$$\frac{1}{4} \int (u+1) \sqrt{u} du = \frac{1}{4} \int (u^{3/2} + u^{1/2}) du$$

$$= \frac{1}{4} \left[\frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C \right]$$

$$= \frac{1}{10} (2x-1)^{5/2} + \frac{1}{6} (2x-1)^{3/2} + C$$

$$(4) \int \cos x \cdot e^{\sin x} dx$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x \quad du = \cos x dx$$

$$\int e^u du = e^u + C = e^{\sin x} + C$$

$$(5) \int \tan^2(2x) dx$$

$$\int (\sec^2(2x) - 1) dx$$

$$\int \sec^2(2x) dx - \int 1 dx$$

$$u = 2x$$

$$\frac{du}{dx} = 2 \quad \frac{1}{2} du = dx$$

$$\frac{1}{2} \int \sec^2 u du$$

$$\frac{1}{2} \tan u$$

$$\frac{\sin^2 \theta + \cos^2 \theta = 1}{\cos^2 \theta} \quad \frac{1}{\cos^2 \theta}$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\boxed{\frac{1}{2} \tan(2x) - x + C}$$