

## NOTES-- Inverses

- ❖  $f$  and  $g$  are inverses of one another if ...

$$f(g(x)) = g(f(x)) = x$$

they are reflected over the line  $y=x$

- ❖ notation: the inverse of  $f(x)$  is ...

$$f^{-1}(x)$$

- ❖ to graph an inverse ...

Switch the  $x$ - and  $y$ - coordinates

- ❖ to determine if a relation is "one-to-one" ...

use the HLT

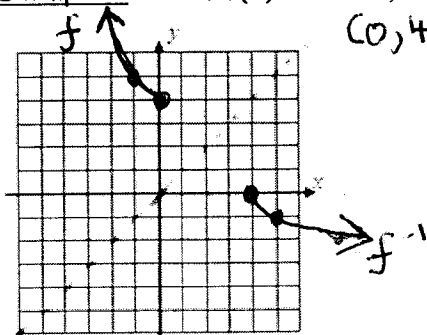
→ the inverse is a function

Example 1 Are  $f(x) = \sqrt{5-x}$  and  $g(x) = 5-x^2, x \geq 0$  inverses?

$$f(g(x)) = \sqrt{5 - (5-x^2)} = \sqrt{5-5+x^2} = \sqrt{x^2} = x$$

$$g(f(x)) = 5 - (\sqrt{5-x})^2 = 5 - (5-x) = 5-5+x = x$$

Example 2 If  $f(x) = x^2 + 4, x \leq 0$ , find  $f^{-1}(x)$ . Sketch  $f$  and  $f^{-1}$  on the same axes.

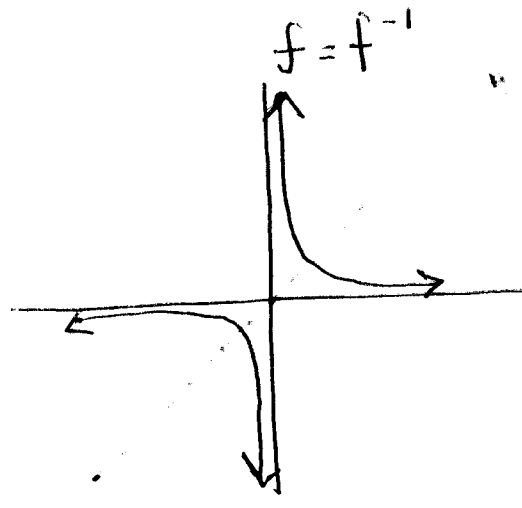


$$\begin{aligned} x &= y^2 + 4 \\ x - 4 &= y^2 \\ y &= \pm \sqrt{x-4} \end{aligned}$$

$$f^{-1}(x) = -\sqrt{x-4}, x \geq 4$$

Example 3 If  $f(x) = \frac{7}{x}$ , find  $f^{-1}(x)$ .

$$x = \frac{7}{y}$$
$$xy = 7$$
$$y = \frac{7}{x}$$
$$f^{-1}(x) = \frac{7}{x}$$



Example 4 Determine if each relation is one-to-one.

