L'Hopital's Rule

Suppose that $f(a)=0$ and $g(a)=0$, that $f'(a)$ & $g'(a)$ exist, and $g'(a) \neq 0$.

Then $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

indeterminate forms: $\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$

why it works...

$$\frac{f(x)}{g(x)} = \frac{f(x)-f(a)}{g(x)-g(a)} = \frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}}$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}} = \frac{f'(a)}{g'(a)}$$

1. $\lim_{x \to 0} \frac{x - \sin x}{x^3}$ $\quad \frac{0-\sin 0}{0^3} = \frac{0}{0}$

$\lim_{x \to 0} \frac{1-\cos x}{3x^2}$ $\quad \frac{1-\cos 0}{3(0)^2} = \frac{0}{0}$

$$\lim_{x \to 0} \frac{\sin x}{6x} \qquad \frac{\sin 0}{6(0)} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\cos x}{6} = \boxed{\frac{1}{6}}$$

other indeterminate forms:

$$\left.\begin{array}{c} \infty \cdot 0 \\ \infty - \infty \end{array}\right\} \text{rewrite}$$

$$\left.\begin{array}{c} 1^{\infty} \\ 0^{0} \\ \infty^{0} \end{array}\right\} \text{use ln then rewrite}$$

* If $\lim_{x \to a} \ln f(x) = L$, then $\lim_{x \to a} f(x) = e^{L}$.

$\ln a = \#$

$e^{\#} = a$

② $\lim\limits_{x\to\infty} x \cdot \tan\frac{1}{x}$ $\quad \infty \cdot 0$

$$\lim_{x\to\infty} \frac{\tan\frac{1}{x}}{\frac{1}{x}} \quad \frac{0}{0}$$

$$\lim_{x\to\infty} \frac{\sec^2\left(\frac{1}{x}\right) \cdot \cancel{-x^{-2}}}{\cancel{-x^{-2}}}$$

$$\lim_{x\to\infty} \sec^2\frac{1}{x} = (1)^2 = \boxed{1}$$

③ $\lim\limits_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = \boxed{e^1}$ $\quad (1+0)^\infty = 1^\infty$

$$\lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty} x \cdot \ln\left(1+\frac{1}{x}\right) \quad \infty \cdot 0$$

$$= \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \quad \frac{0}{0}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}} \cdot \cancel{-x^{-2}}}{\cancel{-x^{-2}}} = \frac{1}{1+0} = 1$$

④ $\lim_{x\to 0^+} (\sin x)^x = e^0$ $\quad 0^0$

$= \boxed{1}$

$$\lim_{x\to 0^+} \ln(\sin x)^x = \lim_{x\to 0^+} x\cdot \ln(\sin x) \qquad 0\cdot -\infty$$

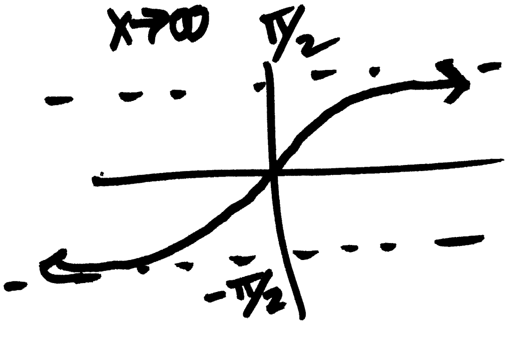
$$= \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}} \qquad \frac{-\infty}{\infty}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cdot \cos x}{-x^{-2}} \to \frac{\cot x}{-\frac{1}{x^2}} \to -x^2\cdot \cot x$$

$$= \lim_{x\to 0^+} \frac{-x^2}{\tan x} \qquad \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{-2x}{\sec^2 x} = \frac{0}{1} = 0$$

⑤ $\lim_{x\to\infty} x^2\cdot \arctan x$ $\qquad \infty\cdot\frac{\pi}{2}$ not indet.



$= \boxed{\infty}$