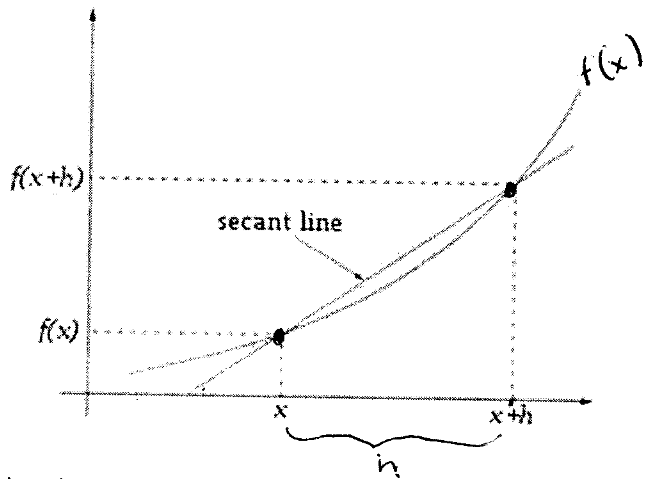
Notes--Limit Definition of a Derivative



$(x, f(x))$ $(x+h, f(x+h))$

the slope of the secant = $\dfrac{f(x+h) - f(x)}{x+h-x} = \dfrac{f(x+h) - f(x)}{h}$ difference quotient

- A **secant line** to the graph of f must intersect it in at least two distinct points.
- A **tangent line** only need intersect the graph in one point, where the line might "just touch" the graph. (There could be other intersection points).

- We define the **slope** of the tangent line to be the (two-sided) **limit** of the difference quotient as $h \to 0$, if that limit exists.
- We denote this slope by $f'(x)$, read as "***f*** **prime** of x".

Definition of the Derivative

the slope of the tangent line

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

provided the limit exists

<u>Examples</u> Use the definition of the derivative to find the derivative of each function with respect to x.

1. $f(x) = -5x + 3$

$$f'(x) = \lim_{h \to 0} \frac{-5(x+h)+3 - (-5x+3)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-5x} - 5h \cancel{+3} \cancel{+5x} \cancel{-3}}{h}$$

$$= \lim_{h \to 0} (-5)$$

$$= \boxed{-5}$$

2. $f(x) = 2x^2 + 7x - 1$

$$\lim_{h\to 0} \frac{2(x+h)^2 + 7(x+h) - 1 - (2x^2+7x-1)}{h}$$

$$\lim_{h\to 0} \frac{\overbrace{2(x^2+2hx+h^2)}^{2x^2+4hx+2h^2} + \cancel{7x} + 7h \cancel{-1} \cancel{-2x^2} \cancel{-7x} \cancel{+1}}{h}$$

$$\lim_{h\to 0} \frac{4hx + 2h^2 + 7h}{h} = \lim_{h\to 0} (4x + 2h + 7) = 4x + 7$$

3. $f(x) = \sqrt{3 - 2x}$

$$\lim_{h\to 0} \frac{(\sqrt{3-2(x+h)} - \sqrt{3-2x})(\sqrt{3-2(x+h)} + \sqrt{3-2x})}{(h)(\sqrt{3-2(x+h)} + \sqrt{3-2x})}$$

$$\lim_{h\to 0} \frac{3-2(x+h) - (3-2x)}{h(\sqrt{3-2(x+h)} + \sqrt{3-2x})} = \lim_{h\to 0} \frac{\cancel{3} - \cancel{2x} - 2h \cancel{-3} + \cancel{2x}}{h(\sqrt{3-2(x+h)} + \sqrt{3-2x})}$$

$$\lim_{h\to 0} \frac{-2}{\sqrt{3-2(x+h)} + \sqrt{3-2x}} = \frac{-2}{\sqrt{3-2x} + \sqrt{3-2x}} = \frac{-\cancel{2}}{\cancel{2}\sqrt{3-2x}}$$

$$= \frac{-1}{\sqrt{3-2x}}$$

4. $f(x) = \frac{2}{(x-5)}$

$$\lim_{h\to 0} \frac{\frac{2(x-5)}{(x-5)(x+h-5)} - \frac{2(x+h-5)}{(x-5)(x+h-5)}}{h} = \lim_{h\to 0} \frac{\frac{2x-10-\cancel{2x}-2h+\cancel{10}}{(x+h-5)(x-5)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-2h}{(x+h-5)(x-5)}}{\frac{h}{1}} = \lim_{h\to 0} \frac{-2\cancel{h}}{(x+h-5)(x-5)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-2}{(x-5)(x-5)} = \frac{-2}{(x-5)^2}$$

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