Limits (Analytically)

① direct substitution

Ⓐ $\lim_{x \to 2} (5x+4) = 5(2)+4 = 14$

Ⓑ $\lim_{x \to 1} \frac{-5}{2x+6} = \frac{-5}{2(1)+6} = \frac{-5}{8}$

$\frac{0}{0}$ "indeterminate" $\Rightarrow$ do algebra

② factor

Ⓐ $\lim_{x \to 2} \frac{x-2}{x^2-x-2} \qquad \frac{2-2}{4-2-2} = \frac{0}{0}$

$\lim_{x \to 2} \frac{x-2}{(x-2)(x+1)} = \lim_{x \to 2} \frac{1}{x+1} = \frac{1}{2+1} = \frac{1}{3}$

Ⓑ $\lim_{x \to 4} \frac{x^2-10x+24}{x^2-2x-8} \qquad \frac{16-40+24}{16-8-8} = \frac{0}{0}$

$\lim_{x \to 4} \frac{(x-6)(x-4)}{(x-4)(x+2)} = \lim_{x \to 4} \frac{x-6}{x+2} = \frac{-2}{6} = \frac{-1}{3}$

(c) $\lim_{x\to 2} \dfrac{x^5-32}{x-2}$ $\qquad \dfrac{32-32}{2-2} = \dfrac{0}{0}$

$\lim_{x\to 2} \dfrac{(x-2)(x^4+2x^3+4x^2+8x+16)}{x-2}$

$$\begin{array}{r|rrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & -32 \\ & \downarrow & 2 & 4 & 8 & 16 & 32 \\ \hline & 1 & 2 & 4 & 8 & 16 & 0 \end{array}$$

$\lim_{x\to 2} (x^4+2x^3+4x^2+8x+16)$

$= 16+16+16+16+16$

$= 80$

(3) common denominator

$\lim_{x\to 0} \dfrac{\frac{1}{x+5} - \frac{1}{5}}{x}$ $\qquad \dfrac{\frac{1}{5}-\frac{1}{5}}{0} = \dfrac{0}{0}$

$\lim_{x\to 0} \dfrac{\frac{5-(x+5)}{5(x+5)}}{x} = \lim_{x\to 0} \dfrac{\frac{5-x-5}{5(x+5)}}{x} = \lim_{x\to 0} \dfrac{\frac{-x}{5(x+5)}}{x}$

$= \lim_{x\to 0} \dfrac{-x}{5(x+5)} \cdot \dfrac{1}{x} = \lim_{x\to 0} \dfrac{-1}{5(x+5)} = \dfrac{-1}{25}$

AP Calculus AB

Notes: The Conjugate Method/Trig Limits

Recall from Precalc that the conjugate of $a + bi$ is $a - bi$ The conjugate of $x-\sqrt{2}$ is $x+\sqrt{2}$.

$\frac{2-2}{0} = \frac{0}{0}$

1) How can the conjugate help us? When trying to evaluate $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$, we multiply by the conjugate of $\sqrt{x+4}-2$ which is $\sqrt{x+4}+2$. Watch what happens:

$$\lim_{x\to 0}\left(\frac{\sqrt{x+4}-2}{x}\right)\left(\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right) = \lim_{x\to 0}\frac{x+4+2\sqrt{x+4}-2\sqrt{x+4}-4}{x(\sqrt{x+4}+2)} = \lim_{x\to 0}\frac{1}{\sqrt{x+4}+2}$$

$$\frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

More examples:

2) $\lim_{t\to 0}\left(\frac{\sqrt{2-t}-\sqrt{2}}{t}\right)\left(\frac{\sqrt{2-t}+\sqrt{2}}{\sqrt{2-t}+\sqrt{2}}\right)$ =

$$\lim_{t\to 0}\frac{2-t-2}{t(\sqrt{2-t}+\sqrt{2})} = \lim_{t\to 0}\frac{-1}{\sqrt{2-t}+\sqrt{2}} = \frac{-1}{\sqrt{2}+\sqrt{2}} = \frac{-1}{2\sqrt{2}}$$

3) $\cancel{\lim_{x\to 0}\frac{\sqrt{2x+3}-\sqrt{3}}{x}}$ =

4) $\lim_{h\to 0}\left(\frac{h}{\sqrt{3h+7}-\sqrt{7}}\right)\left(\frac{\sqrt{3h+7}+\sqrt{7}}{\sqrt{3h+7}+\sqrt{7}}\right)$

$$\lim_{h\to 0}\frac{h(\sqrt{3h+7}+\sqrt{7})}{3h+7-7}$$

$$\lim_{h\to 0}\frac{\sqrt{3h+7}+\sqrt{7}}{3} = \boxed{\frac{2\ldots}{3}}$$

Evaluating limits involving Trig:

1) $\lim_{x\to\pi}\frac{\sin x+\cos x}{2\cos x} =$ _____

2) $\lim_{\theta\to 0}\sin\theta\cos\theta =$ _____

3) $\lim_{\theta\to\frac{5\pi}{6}}\sin\theta\cos\theta =$ _____

4) $\lim_{\theta\to\frac{\pi}{2}}\tan\theta\cos^2\theta\sec\theta =$ _____

5) $\lim_{x\to\pi}\frac{1}{4\cos x} =$ _____

6) $\lim_{\theta\to 0}\tan\theta =$ _____

7) $\lim_{\theta\to\frac{3\pi}{4}}\cot\theta =$ _____

8) $\lim_{\theta\to\frac{-\pi}{2}}\sin\theta\cot\theta =$ _____