Limits

4 ways to represent

1. Numerically (table of values)
2. Graphically
3. Analytically (algebra)
4. Verbally

End Behavior what y-value (if there is one) a function approaches as $x \to \infty$ or $-\infty$

horizontal asymptote

If $\lim\limits_{x \to \pm\infty} f(x) = L$ ← y-value

then $y = L$ is a h.a.

EX 1 Find the horiz. asympt.

(A) $f(x) = \dfrac{6x^3 + 2x + 1}{4x^3 + 1}$ $\quad y = \dfrac{6}{4} = \dfrac{3}{2}$ h.a.

(B) $f(x) = \dfrac{3x - 2}{\sqrt{\underbrace{2x^2}_{\text{more important}} + 1}}$ $\quad \sqrt{2x^2}$ $\quad \sqrt{2} \cdot \sqrt{x^2}$ $\quad \sqrt{2}\,|x|$

h.a. $y = \dfrac{3}{\sqrt{2}}$ and $y = -\dfrac{3}{\sqrt{2}}$

(C) $f(x) = \dfrac{3x^4 + 2x^2 + 5}{1 + 4x^5}$ h.a. $y = 0$

(D) $f(x) = \dfrac{1 + 4x^5}{3x^4 + 2x^2 + 5}$ no h.a.

EX2 Find each limit.

(A) $\lim\limits_{x \to \infty} \left(5 - \dfrac{2}{x^2}\right) = 5 - 0 = \boxed{5}$

(B) $\lim\limits_{x \to \infty} \dfrac{2x - 1}{x + 1} = \dfrac{2}{1} = \boxed{2}$

(C) $\lim\limits_{x \to -\infty} \dfrac{4x^2 - 1}{5 + 3x^2} = \boxed{\dfrac{4}{3}}$

(D) $\lim\limits_{x \to \infty} \dfrac{4x - 1}{3x^2 + 5} = \boxed{0}$

(E) $\lim\limits_{x \to -\infty} \dfrac{4x^3 - 1}{3x^2 + 5}$ DNE, direction $\boxed{-\infty}$