Notes--One-sided Limits & Continuity

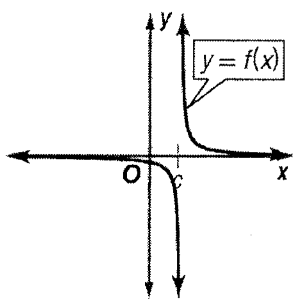
Continuity at a Point
f(x) is continuous at $x = c$, if and only if, $\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = f(c)$.

Continuity on an Open Interval
f(x) is continuous on the interval (a,b), if and only if, f(x) is continuous at all $x \in (a,b)$.

Key Concept — Types of Discontinuity

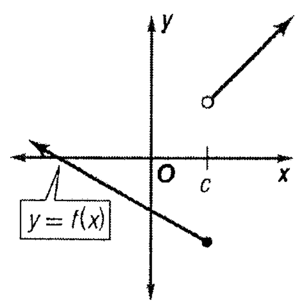
A function has an **infinite discontinuity** at $x = c$ if the function value increases or decreases indefinitely as x approaches c from the left and right.

Example



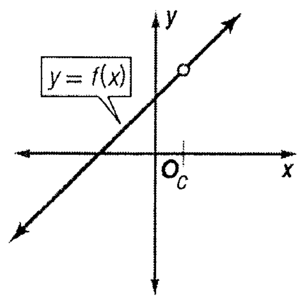
A function has a **jump discontinuity** at $x = c$ if the limits of the function as x approaches c from the left and right exist but have two distinct values.

Example



A function has a **removable discontinuity** if the function is continuous everywhere except for a hole at $x = c$.

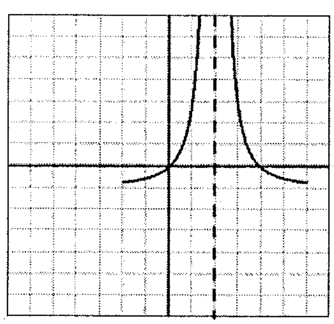
Example



Example 1 Refer to the graph to find each of the following:

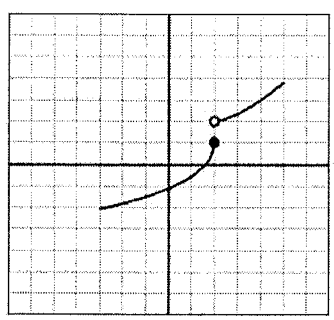
a) the value(s) of x for which the function is discontinuous
b) why it is discontinuous at that value
c) the type of discontinuity
d) whether it is removable (R) or nonremovable (NR) discontinuity

1)



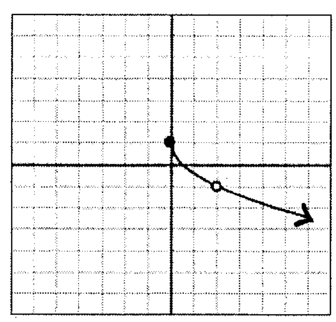
a) $x = 2$
b) $\lim_{x\to2^-} f(x) = \infty$ $\lim_{x\to2^+} f(x) = \infty$
c) infinite
d) NR

2)



a) $x = 2$
b) $\lim_{x\to2^-} f(x) = 1$ $\lim_{x\to2^+} f(x) = 2$
c) jump
d) NR

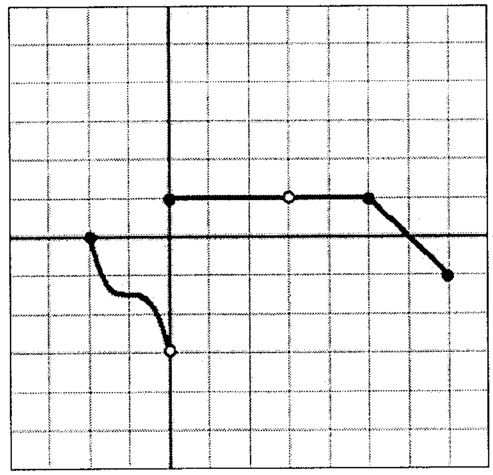
3)



a) $x = 2$
b) continuous everywhere except at $x = 2$
c) hole (removable)
d) R

Example 2

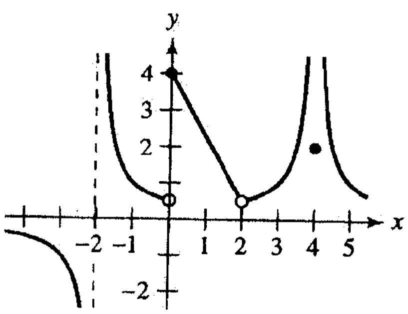
Based on the graph evaluate the following.



1. $\lim_{x\to 0^-} f(x) =$ -3
2. $\lim_{x\to 0^+} f(x) =$ 1
3. $\lim_{x\to 0} f(x) =$ DNE
4. $\lim_{x\to 1^-} f(x) =$ 1
5. $\lim_{x\to 1^+} f(x) =$ 1
6. $\lim_{x\to 1} f(x) =$ 1
7. $\lim_{x\to 5} f(x) =$ 1
8. $f(1) =$ 1
9. $f(0) =$ 1
10. $f(-2) =$ 0
11. $\lim_{x\to 6^-} f(x) =$ 0
12. $\lim_{x\to 6^-} f(x) =$ 0
13. $\lim_{x\to 6} f(x) =$ 0
14. $f(6) =$ 0
15. $\lim_{x\to 3} f(x) =$ 1
16. $f(3) =$ DNE
17. $\lim_{x\to -1} f(x) \approx$ -1.5
18. $f(-1) \approx$ -1.5
19. True or False: $\lim_{x\to c} f(x)$ exists at every c on (1,3)   limit = 1
20. True or False: $\lim_{x\to c} f(x)$ exists at every c on (−2,1)

DNE at x=0

Example 3 Use the graph of f(x) below to find the following:



$f(0) =$ 4 $f(2) =$ DNE $\lim_{x\to 2^+} f(x) = \frac{1}{2}$ $\lim_{x\to 2^-} f(x) = \frac{1}{2}$

$\lim_{x\to 2} f(x) = \frac{1}{2}$ $\lim_{x\to 0^-} f(x) = \frac{1}{2}$ $\lim_{x\to 0^+} f(x) = 4$ $\lim_{x\to 0} f(x) =$ DNE

$\lim_{x\to -2^-} f(x) = -\infty$ $\lim_{x\to -2^+} f(x) = \infty$ $\lim_{x\to -2} f(x) =$ DNE

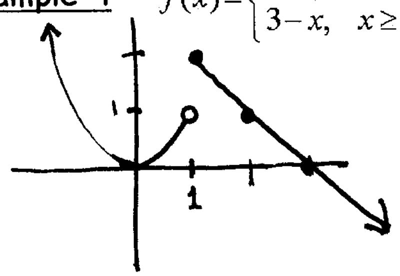
Example 4

$$f(x)=\begin{cases} x^2, & x<1 \\ 3-x, & x\ge 1 \end{cases}$$

Find $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 0^-} f(x)$.

0   0

$\lim_{x\to 0} f(x) = 0$



$\lim_{x\to 1^+} f(x) = 2$ $\lim_{x\to 1^-} f(x) = 1$ $\lim_{x\to 1} f(x) =$ DNE

Example 5 **Evaluate each limit.**

a. $\lim_{x\to 2^+} \frac{x^2-4}{x-2} = \lim_{x\to 2^+} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$

$= 4$ hole when $x=2$

b. $\lim_{x\to -1^-} \frac{x^2-1}{x^3+1} = \lim_{x\to -1^-} \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(x^2-x+1)}$

$= \frac{-2}{1+1+1} = -\frac{2}{3}$

c. $\lim_{x\to 1^+} (2x+3) = 5$

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