Notes--Limits Approaching a Real Number

Definition of a Limit

$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

y-value   from the left side   from the right side

Existence of a Limit

$\lim_{x \to c} f(x) = L$ **if and only if**

$\lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^-} f(x) = L$

What this says is that if you don't get closer and closer to a number (the "*y*" or $f(x)$) from both sides of the "*x*", then there is no **limit** of $f(x)$ at that point.

Now the actual point $f(c)$ may be defined (as in a non-continuous function) at a completely different *y* (where no limit may occur), but in order for a limit to occur, the *x*'s have to approach a certain *y* value **from both sides.**

Three Ways to Find a Limit:

1) use a graph
2) algebraically
3) numerically (table)

Strategies for finding a limit approaching a real number

1. substitute $x = a$

- if you get a number, that is the limit value
- if you get $\frac{\#}{0}$, the limit does not exist (the limit might have a direction)
- if you get $\frac{0}{0}$, do more work and then evaluate the limit
  - factor/reduce
  - find a common denominator
  - simplify complex fractions
  - multiply by the conjugate of an expression with a radical

2. if piecewise: check to see that the right-hand limit = left-hand limit

Directions: Find each limit.

1. $\lim_{x \to 2} (4x^3) = 4(2)^3 = \boxed{32}$

2. $\lim_{x \to 3} \frac{x-1}{x^2-1} = \frac{3-1}{3^2-1} = \frac{2}{8} = \boxed{\frac{1}{4}}$

3. $\lim\limits_{x\to 1}\dfrac{x-1}{x^2-1}$ $\quad \dfrac{1-1}{1^2-1}=\dfrac{0}{0}$

$$\lim_{x\to 1}\frac{\cancel{x-1}}{(\cancel{x-1})(x+1)}=\lim_{x\to 1}\frac{1}{x+1}=\frac{1}{1+1}=\boxed{\frac{1}{2}}$$

4. $\lim\limits_{x\to -2}\dfrac{x^2-2x-8}{x^2-4}$ $\quad \dfrac{(-2)^2-2(-2)-8}{(-2)^2-4}=\dfrac{0}{0}$

$$\lim_{x\to -2}\frac{(x-4)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}=\lim_{x\to -2}\frac{x-4}{x-2}=\frac{-2-4}{-2-2}=\frac{-6}{-4}=\boxed{\frac{3}{2}}$$

5. $\lim\limits_{x\to 0}\dfrac{(x-2)^2-4}{x}$ $\quad \dfrac{(0-2)^2-4}{0}=\dfrac{0}{0}$

$$\lim_{x\to 0}\frac{x^2-4x\cancel{+4-4}}{x}=\lim_{x\to 0}\frac{\cancel{x}(x-4)}{\cancel{x}}=\lim_{x\to 0}(x-4)=0-4=\boxed{-4}$$

6. $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}$ $\quad \dfrac{\sqrt{9}-3}{9-9}=\dfrac{0}{0}$

$$\lim_{x\to 9}\left(\frac{\sqrt{x}-3}{x-9}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)=\lim_{x\to 9}\frac{x\cancel{+3\sqrt{x}-3\sqrt{x}}-9}{(x-9)(\sqrt{x}+3)}=\lim_{x\to 9}\frac{\cancel{x-9}}{(\cancel{x-9})(\sqrt{x}+3)}$$

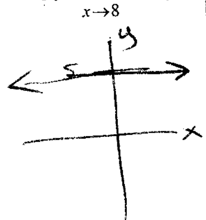
$$=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}=\boxed{\frac{1}{6}}$$

7. $\lim\limits_{x\to 4}\dfrac{\sqrt{x+5}-3}{x-4}$ $\quad \dfrac{\sqrt{9}-3}{4-4}=\dfrac{0}{0}$

$$\lim_{x\to 4}\left(\frac{\sqrt{x+5}-3}{x-4}\right)\left(\frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}\right)=\lim_{x\to 4}\frac{\overset{x-4}{\cancel{x+5\cancel{+3\sqrt{x+5}-3\sqrt{x+5}}-9}}}{(\cancel{x-4})(\sqrt{x+5}+3)}=\lim_{x\to 4}\frac{1}{\sqrt{x+5}+3}$$

$$=\frac{1}{\sqrt{4+5}+3}=\boxed{\frac{1}{6}}$$

8. $\lim\limits_{x\to 8}5=\boxed{5}$



9. $\lim\limits_{x\to 5}\dfrac{x+1}{x^2-25}$ $\quad \dfrac{5+1}{5^2-25}=\dfrac{6}{0}$ $\quad \boxed{\text{does not exist}}$