NOTES--LOGARITHMIC FUNCTIONS

A logarithmic function has form $f(x) = \log_a x$ where $a > 0$ and $a \neq 1$

if a = 10, it is a "common log"

if a = e, it is a "natural log"

Convert to exp. form

$a^y = x$

Example 1 Evaluate each expression.

A. $\log_3 9 = 2$

$3^y = 9$

$y = 2$

B. $\log_2 64 = 6$

$2^y = 64$

$y = 6$

C. $\log_3\left(\frac{1}{27}\right) = -3$

$3^y = \frac{1}{27}$

$y = -3$

D. $\log_8 \sqrt[4]{8} = \frac{1}{4}$

$8^y = \sqrt[4]{8} = 8^{1/4}$

$y = \frac{1}{4}$

E. $\log_2 \sqrt[3]{32} = \frac{5}{3}$

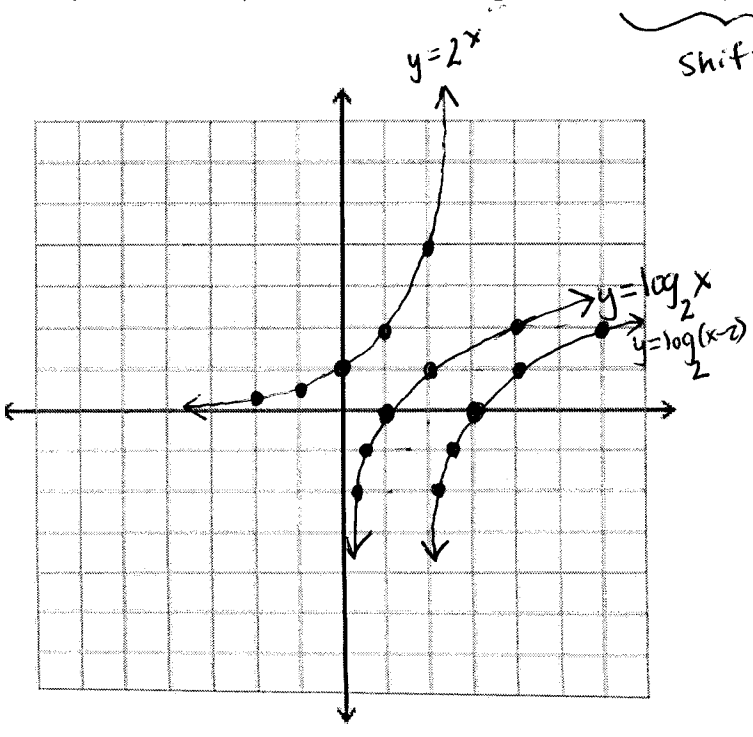
$2^y = \sqrt[3]{32} = 32^{\frac{1}{3}} = (2^5)^{\frac{1}{3}} = 2^{\frac{5}{3}}$

$y = \frac{5}{3}$

change of base

$\log_3 9 = \frac{\log 9}{\log 3}$

Example 2 Graph $y = 2^x$, $y = \log_2 x$, and $y = \log_2(x-2)$ on the same set of axes.



shift right 2

$y = 2^x$

x	y
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

switch!

$y = \log_2 x$

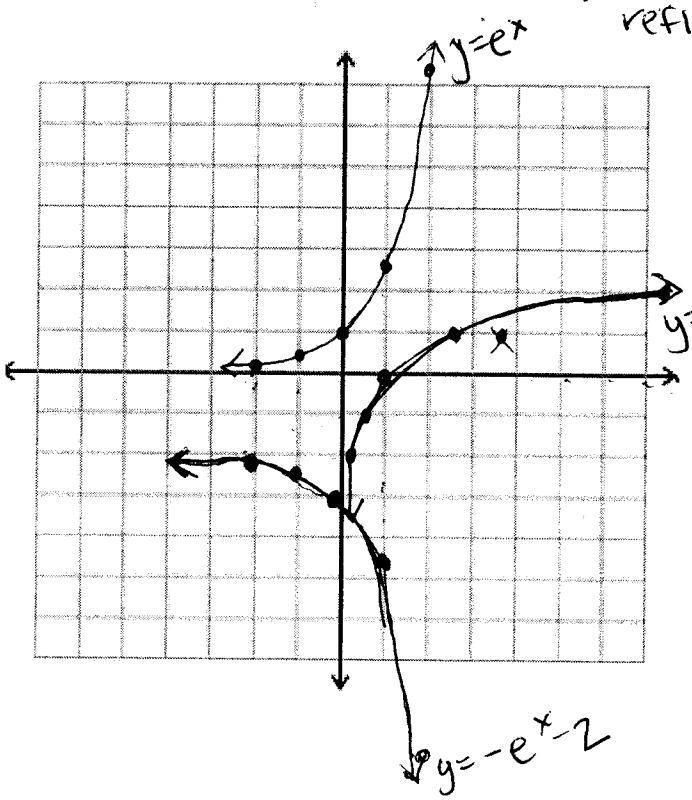
$x = \log_2 y$ } inverses

$2^x = y$

switch x- and y-coords!

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2

Example 3 Graph $y = e^x$, its inverse, and $y = -e^x - 2$ on the same set of axes.



refl. over x-axis, down 2

$y = e^x$

x	y
−2	$e^{-2} = .135$
−1	.368
0	1
1	$e^1 = 2.718$
2	$e^2 = 7.389$

inverse of $y = e^x$

$x = e^y$

$\log_e x = y$

$\ln x = y$

.135	−2
.368	−1
1	0
2.718	1
7.389	2

Example 4 Let $N = P\left(1 - e^{-0.15d}\right)$ where N = the number of people who have heard a rumor, P is the total population, and d is the number of days elapsed since the rumor began. In a community of 1000 students, how many days have elapsed before 450 students have heard the rumor?

$y_1 \qquad y_2$

$450 = 1000\left(1 - e^{-.15d}\right)$

intersect when $d = 3.98558$

almost 4 days