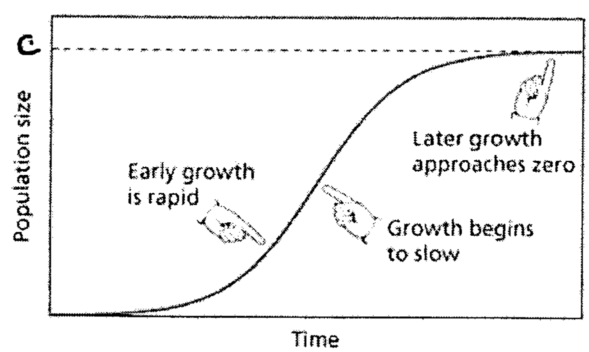
# NOTES--LOGISTIC FUNCTIONS

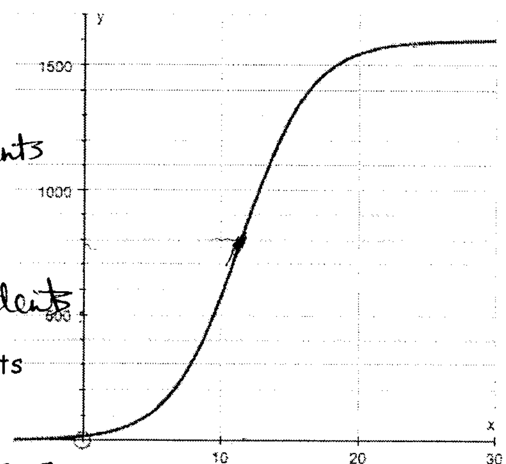
**Logistic growth functions**—functions that model situations where exponential growth is limited.

- form: $\mathbf{p(t)} = \frac{\mathbf{c}}{\mathbf{1+ae^{-bt}}}$ where a, b, and c are constants with $c > 0$ *and* $b > 0$.
- c is the carrying Capacity because the value of p(t) approaches c as t approaches infinity $\lim_{t \to \infty} p(t) = c$
- The graph of a logistic function looks like an exponential function at first, but then it levels off at y = c. The logistic function has two horizontal asymptotes: y = 0 and y = c.

**Logistic Curve**




**Example 1** The number of students infected with flu after t days at Holly Springs High School is modeled by the following function: $p(t) = \frac{1600}{1+99e^{-0.4t}}$




a) What was the initial number of infected students?

$p(0) = \frac{1600}{1+99e^{0}} = \frac{1600}{100} = 16$ students

b) After 5 days, how many students will be infected?

$p(5) = 111$ students

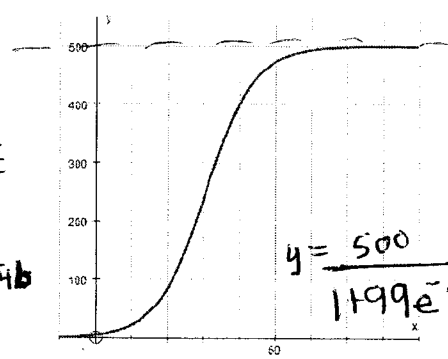
c) What is the maximum number of students that will be infected?

carry. cap. = 1600 students

d) According to the model, when will the number of students infected be 800?

$800 = \frac{1600}{1+99e^{-.4t}}$ $t = 11.5$ days

**Example 2** Find a logistic equation of the form $y = \frac{c}{1+ae^{-bt}}$ that fits the given graph, if the y-intercept is 5 and the point (24, 135) is on the curve.




(0, 5) carrying cap. = 500 = c

$y = \frac{500}{1+ae^{-bt}}$

$5 = \frac{500}{1+ae^{-b(0)}}$

$\frac{5}{1} = \frac{500}{1+a}$

$5 + 5a = 500$

$5a = 495$

$a = 99$

$y = \frac{500}{1+99e^{-bt}}$

$135 = \frac{500}{1+99e^{-24b}}$

$b = .15$

$y = \frac{500}{1+99e^{-.15t}}$

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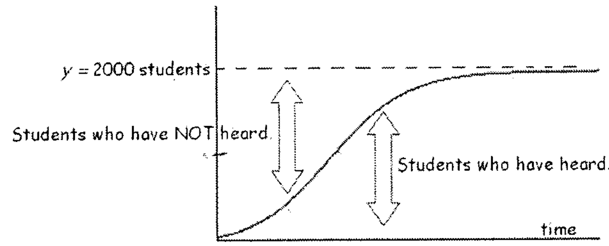
**Example 3** A horrible rumor begins to spread within HSHS. Lilly, Tucker, and Carter begin telling people that "Ms. Powell does not have any friends and lives a boring life". After t days, the number of persons who have heard this rumor is modeled by the logistic function

$$R(t) = \frac{2000}{1+115e^{-0.97t}}.$$




a) How many people on campus originally heard the rumor? Round to the nearest whole person.

$R(0) = \frac{2000}{1+115e^{0}} = \frac{2000}{116} = 17$ people

b) To the nearest whole person, find the number of people who have heard the rumor after 1 day, after 2 days, and after 5 days. $R(1) = 44$ people $R(2) = 114$ people $R(5) = 1052$ people

c) How many people have heard the rumor when the rumor is growing the fastest? On which day did this occur? $\frac{1}{2}(2000) = 1000$ people. $1000 = \frac{2000}{1+115e^{-.97t}}$ $t = 4.89$ on the 5th day

d) Describe the end behavior of the graph. What does this mean in terms of the rumor?

$\lim_{t \to \infty} R(t) = 2000$ At most about 2000 people will hear the rumor.

**Example 4** (modeling data with a logistic function) The data in the table represents the amount of yeast biomass present after t hours in a culture.

| Time (in hours) | Yeast Biomass | Time (in hours) | Yeast Biomass |
|---|---|---|---|
| 0 | 9.6 | 10 | 513.3 |
| 1 | 18.3 | 11 | 559.7 |
| 2 | 29.0 | 12 | 594.8 |
| 3 | 47.2 | 13 | 629.4 |
| 4 | 71.1 | 14 | 640.8 |
| 5 | 119.1 | 15 | 651.1 |
| 6 | 174.6 | 16 | 655.9 |
| 7 | 257.3 | 17 | 659.6 |
| 8 | 350.7 | 18 | 661.8 |
| 9 | 441.0 | | |

a) Find a logistic model for the data. $y = \frac{663.022}{1+71.576e^{-.547t}}$

b) What is the predicted carrying capacity of the culture? 663.022

c) Predict the population of the culture at 20 hours. $P(20) = 662.181$