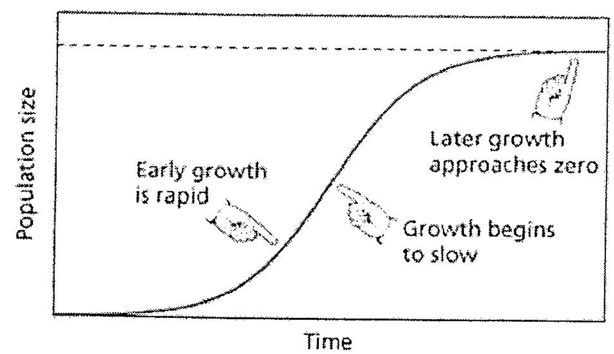
NOTES--LOGISTIC FUNCTIONS

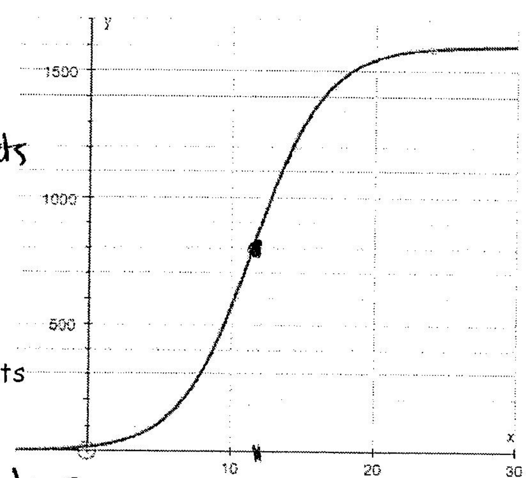
__Logistic growth functions__—functions that model situations where exponential growth is limited.

- form: $\mathbf{p(t)} = \frac{c}{1+ae^{-bt}}$ where a, b, and c are constants with $c > 0$ *and* $b > 0$.
- c is the carrying capacity because the value of p(t) approaches c as t approaches infinity $\lim_{t\to\infty} p(t) = c$
- The graph of a logistic function looks like an exponential function at first, but then it levels off at y = c. The logistic function has two horizontal asymptotes: y = 0 and y = c.

Logistic Curve



__Example 1__ The number of students infected with flu after t days at Holly Springs High School is modeled by the following function: $p(t) = \frac{1600}{1+99e^{-0.4t}}$



a) What was the initial number of infected students?

$p(0) = \frac{1600}{1+99e^{-.4(0)}} = \frac{1600}{100} = 16$ students

b) After 5 days, how many students will be infected?

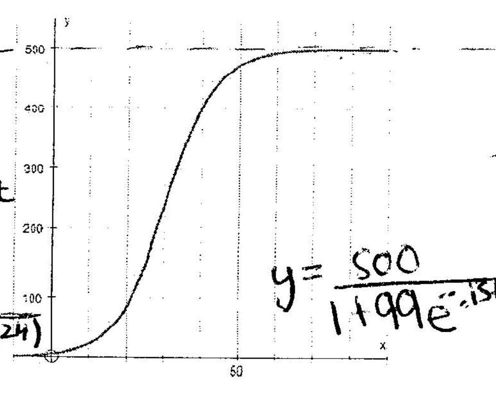
$P(5) = \frac{1600}{1+99e^{-.4(5)}} = 111$ students

c) What is the maximum number of students that will be infected? carrying capacity = 1600

d) According to the model, when will the number of students infected be 800?

$800 = \frac{1600}{1+99e^{-0.4t}}$ t = 11.5 days

__Example 2__ Find a logistic equation of the form $y = \frac{c}{1+ae^{-bt}}$ that fits the given graph, if the __y-intercept is 5__ and the point (24, 135) is on the curve.



$(\overset{t}{0}, \overset{y}{5})$ t y Carrying cap. 500

$y = \frac{500}{1+ae^{-bt}}$

$5 = \frac{500}{1+ae^{-b(0)}}$

$5 = \frac{500}{1+a}$

$\frac{5(1+a)}{5} = \frac{500}{5}$

$1+a = 100$

$y = \frac{500}{1+99e^{-bt}}$

$135 = \frac{500}{1+99e^{-b(24)}}$

$y = \frac{500}{1+99e^{-.13t}}$

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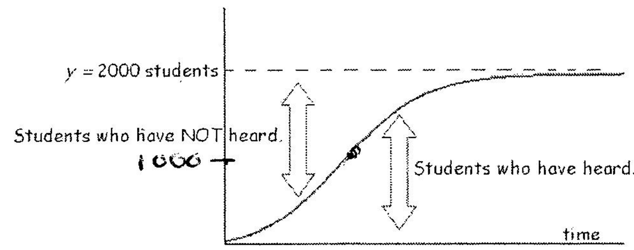
Example 3 A horrible rumor begins to spread within HSHS. Lilly, Tucker, and Carter begin telling people that "Ms. Powell does not have any friends and lives a boring life". After t days, the number of persons who have heard this rumor is modeled by the logistic function

$$R(t) = \frac{2000}{1+115e^{-0.97t}}.$$



a) How many people on campus originally heard the rumor? Round to the nearest whole person.

$R(0) = \frac{2000}{1+115e^{-.97(0)}} = \frac{2000}{116} = 17$ people

b) To the nearest whole person, find the number of people who have heard the rumor after 1 day, after 2 days, and after 5 days. $R(1) = 44$ people $R(2) = 114$ people $R(5) = 1052$ people

c) How many people have heard the rumor when the rumor is growing the fastest? On which day did this occur? $\frac{1}{2}(2000) = 1000$ people; $1000 = \frac{2000}{1+115e^{-.97t}}$ $t = 4.892$ on 5th day

d) Describe the end behavior of the graph. What does this mean in terms of the rumor?

$\lim_{t\to\infty} R(t) = 2000$ At most about 2000 students will hear the rumor.

Example 4 (modeling data with a logistic function) The data in the table represents the amount of yeast biomass present after t hours in a culture.

Time (in hours)	Yeast Biomass	Time (in hours)	Yeast Biomass
0	9.6	10	513.3
1	18.3	11	559.7
2	29.0	12	594.8
3	47.2	13	629.4
4	71.1	14	640.8
5	119.1	15	651.1
6	174.6	16	655.9
7	257.3	17	659.6
8	350.7	18	661.8
9	441.0		

a) Find a logistic model for the data. $y = \frac{663.022}{1+71.576e^{-.547x}}$

b) What is the predicted carrying capacity of the culture? 663.022

c) Predict the population of the culture at 20 hours. $P(20) = 662.181$