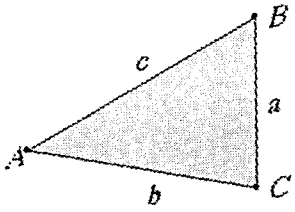
NOTES--Law of Sines and Law of Cosines

Law of Sines



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Sines

AAS ONE SOLUTION

SSA AMBIGUOUS CASE

ASA ONE SOLUTION

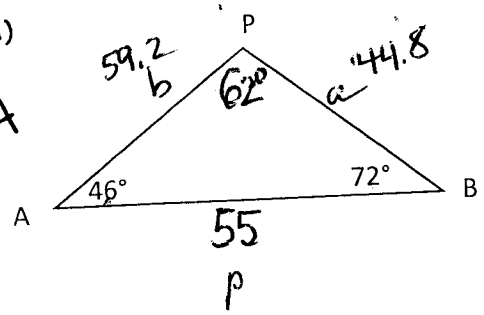
$180° - 1^{st}$ angle

- Domain error — NO SOLUTION
- Second angle option violates triangle angle-sum theorem — ONE SOLUTION
- Both angles satisfy triangle angle-sum theorem — TWO SOLUTIONS

Example 1 Solve the triangle.

A)

ASA



$P = 180° - 46° - 72° = 62°$

$$\frac{\sin 72°}{b} = \frac{\sin 62°}{55} \qquad \frac{\sin 46°}{a} = \frac{\sin 62°}{55}$$

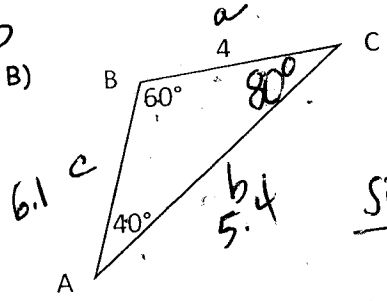
$$\sin 62° \cdot b = 55 \sin 72° \qquad a \cdot \sin 62° = 55 \sin 46°$$

$$b = \frac{55 \sin 72°}{\sin 62°} \qquad a = \frac{55 \sin 46°}{\sin 62°}$$

$$b = 59.2 \qquad a = 44.8$$

AAS

B)



$C = 80°$

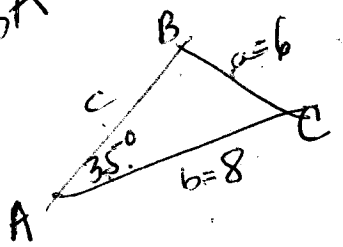
$$\frac{\sin 80°}{c} = \frac{\sin 40°}{4} \qquad \frac{\sin 60°}{b} = \frac{\sin 40°}{4}$$

$$c \cdot \sin 40° = 4 \sin 80° \qquad b \cdot \sin 40° = 4 \sin 60°$$

$$c = \frac{4 \sin 80°}{\sin 40°} = 6.1 \qquad b = \frac{4 \sin 60°}{\sin 40°} = 5.4$$

C) a = 6, b = 8, A = 35°

SSA



$$\frac{\sin B}{8} = \frac{\sin 35^\circ}{6}$$

$$6 \cdot \sin B = 8 \sin 35^\circ$$

$$\sin B = \frac{8 \sin 35^\circ}{6}$$

$$B = \sin^{-1}(\nearrow)$$

$B = 49.9^\circ$ OR $B = 180^\circ - 49.9^\circ = 130.1^\circ$

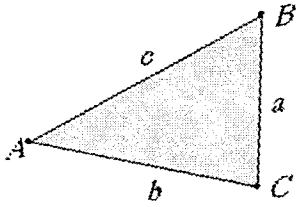
$B = 49.9^\circ$	$B = 130.1^\circ$
$C = 95.1^\circ$	$C = 14.9^\circ$
$c = 10.4$	$c = 2.7$

$$\frac{\sin 95.1^\circ}{c} = \frac{\sin 35^\circ}{6} \qquad \frac{\sin 14.9^\circ}{c} = \frac{\sin 35^\circ}{6}$$

$$c \cdot \sin 35^\circ = 6 \sin 95.1^\circ \qquad c \cdot \sin 35^\circ = 6 \sin 14.9^\circ$$

$$c = \frac{6 \sin 95.1^\circ}{\sin 35^\circ} \quad c = 10.4 \qquad c = \frac{6 \sin 14.9^\circ}{\sin 35^\circ} \quad c = 2.7$$

Law of Cosines



$$a^2 = b^2 + c^2 - 2bc\cos A$$

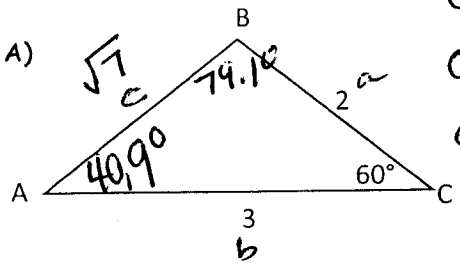
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Example 2

SAS

A)



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 2^2 + 3^2 - 2(2)(3)\cos 60^\circ$$

$$c^2 = 7$$

$$c = \sqrt{7} \approx 2.6$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$3^2 = 2^2 + \sqrt{7}^2 - 2(2)(\sqrt{7})\cos B$$

$$\frac{-2}{-4\sqrt{7}} = \frac{-4\sqrt{7}\cos B}{-4\sqrt{7}}$$

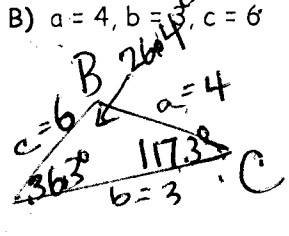
$$B = \cos^{-1}\left(\frac{2}{4\sqrt{7}}\right)$$

$$B = 79.1^\circ$$

$$A = 180^\circ - 60^\circ - 79.1^\circ = 40.9^\circ$$

SSS

B) a = 4, b = 3, c = 6



$$6^2 = 4^2 + 3^2 - 2(4)(3)\cos C$$

$$\frac{11}{-24} = \frac{-24\cos C}{-24}$$

$$C = \cos^{-1}\left(-\frac{11}{24}\right)$$

$$C = 117.3^\circ$$

$$\frac{\sin A}{4} = \frac{\sin 117.3^\circ}{6}$$

$$\frac{6\sin A}{6} = \frac{4\sin 117.3^\circ}{6}$$

$$A = \sin^{-1}(\nearrow)$$

$$A = 36.3^\circ$$

$$B = 180^\circ - 117.3^\circ - 36.3^\circ = 26.4^\circ$$