Notes -- Multiplying Probabilities

Two events are independent if the outcome of one event has no effect on the outcome of the other event.

examples: toss a coin & roll a die

a 3-digit combination with repetition

Two events are dependent if the outcome of one event does effect the outcome of the other event.

examples: a 3-digit combination when digits can't be repeated

Given a jar of marbles: pick one, don't replace, pick another one

> If A and B are successive, independent events:
>
> $p(A \cap B) = p(A) \cdot p(B)$
>
> "and"
>
> If A and B are successive, dependent events:
>
> $p(A \cap B) = p(A) \cdot p(B\ given\ that\ A\ has\ occured)$

Example 1 Toss a coin and roll a die., Find the probability that the coin is tails and the die is a # less than 3.

indep. $p(\text{tails AND } \# < 3) = p(\text{tails}) \cdot p(\# < 3) = \frac{1}{2} \cdot \frac{2}{6} = \frac{1}{6}$

$= .167$

Example 2 Draw a card, replace, draw another card. Find the probability that each card is a diamond.

indep. $p(\text{diam. AND diam.}) = \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{16} = .0625$

Example 3 Draw a card, don't replace, draw another card. Find the probability that each card is a diamond.

dep. $p(\text{diam AND diam}) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17} = .059$

Example 4 Two dice (red and green).

A. Find the probability that the red die shows a "1" and the green die shows a "6".

indep $p(\text{1 on red \& 6 on green}) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

B. Find the probability that the red die shows a # > 4 and the green die shows an even #.

indep. $p(\#>4 \text{ red \& even green}) = \frac{2}{6} \cdot \frac{3}{6} = \frac{1}{6}$

Example 5 Draw a card, don't replace, draw another card. Find the probability that each card is a red.

dep. $p(\text{red AND red}) = \frac{26}{52} \cdot \frac{25}{51} = \frac{25}{102} = .245$

Example 6 5 letters: M, A, T, H, O. Two tiles are chosen randomly. Find each probability:

A. p(selecting 2 vowels) if no replacement occurs

dep $p(\text{vowel AND vowel}) = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10} = .1$

B. p(selecting 2 vowels) if replacement occurs

indep $p(\text{vowel AND vowel}) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = .16$

C. p(selecting the same letter twice) if replacement occurs

indep. $p(\text{M and M}) = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25} * 5 = \boxed{\frac{1}{5}}$

D. p(selecting the same letter twice) if no replacement occurs

dep $p(\text{M and M}) = \frac{1}{5} \cdot 0 = 0$

Conditional Probability

> the probability of event B, assuming that event A has already occurred:
>
> $$p(B|A) = \frac{n(B \cap A)}{n(A)} = \frac{number\ of\ outcomes\ common\ to\ B\ and\ A}{number\ of\ outcomes\ in\ A}$$

"given that"

Example 7 A letter is selected randomly from the letters of the English alphabet. Find the probability of selecting a vowel, given that the outcome is a letter that precedes h.

$$p(\text{vowel} \mid \text{letter before H}) = \frac{2}{7}$$

Example 8 The table shows the differences in political ideology per 100 males and per 100 females in the 2000 US presidential election. Assuming that these numbers are representative of all Americans and one American is randomly selected, find the probability that the person:

A. is liberal, given that the person is a female

$$p(\text{liberal} \mid \text{female}) = \frac{\#\ \text{lib. \& fem.}}{\#\ \text{fem.}} = \frac{20}{100} = \frac{1}{5} = .2$$

B. is male, given that the person is conservative

$$p(\text{male} \mid \text{conserv.}) = \frac{\#\ \text{male \& conserv.}}{\#\ \text{conserv.}} = \frac{39}{64} = .609$$

	Liberal	Moderate	Conservative
Male	16	45	39
Female	20	55	25
			64