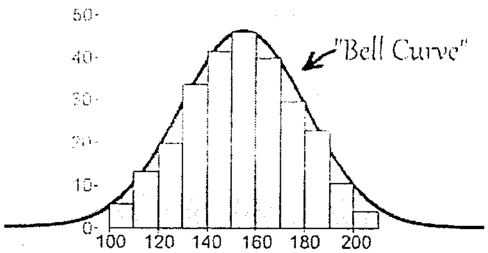
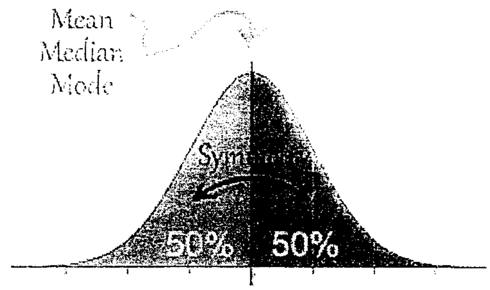
NOTES--Normal Distribution

<u>normal distribution</u>-- where the data tends to be around a central value with no bias left or right, and it gets close to a "Normal Distribution"

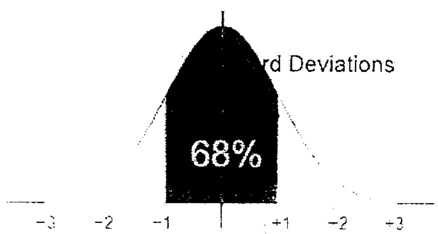


A Normal Distribution

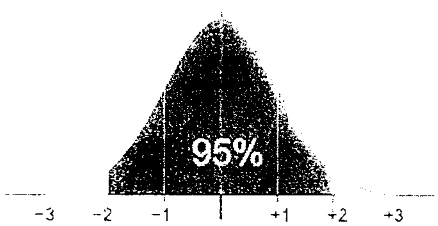


the <u>Normal Distribution</u> has:

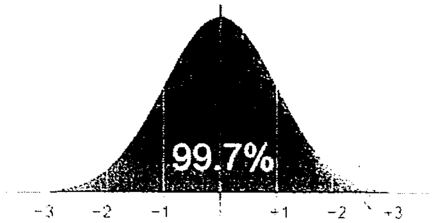
- mean = median = mode
- symmetry about the center
- 50% of values less than the mean and 50% greater than the mean



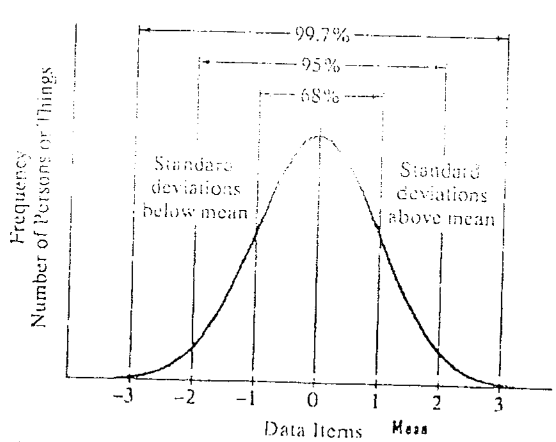
68% of values are within
1 standard deviation of the mean



95% of values are within
2 **standard deviations** of the mean



99.7% of values are within
.3 **standard deviations** of the mean

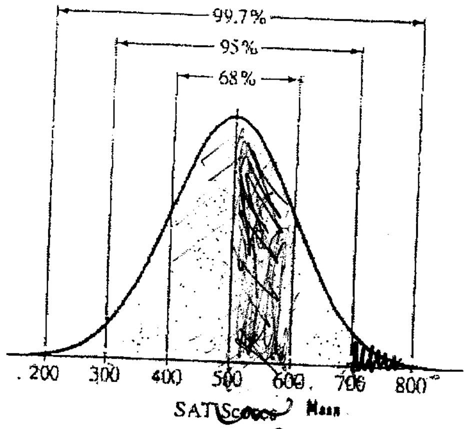


Example 1 With a mean of 500 and a standard deviation of 100, find the SAT score that is

a. 2 standard deviations above the mean. 700

b. 3 standard deviations below the mean. 200

Normal Distribution of SAT Scores



34%

Example 2 Find the percentage of seniors who score:

a. between 400 and 600. 68%

b. between 500 and 600. 34%

c. above 700. $\frac{1}{2}(5\%) = 2.5\%$

Example 3 Find the percentage of seniors who score:

a. between 300 and 700. 95%

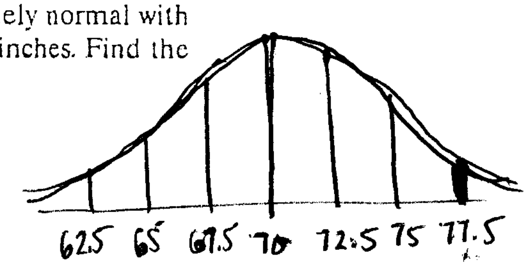
b. between 500 and 700. 47.5%

c. above 600. 50% − 34% = 16%

Example 4 The distribution of heights of young men is approximately normal with a mean of 70 inches and a standard deviation of 2.5 inches. Find the height that is

a. 3 standard deviations above the mean. 77.5

b. 2 standard deviations below the mean. 65



COMPUTING z-SCORES

A z-score describes how many standard deviations a data item in a normal distribution lies above or below the mean. The z-score can be obtained using

$$z\text{-score} = \frac{\text{data item} - \text{mean}}{\text{standard deviation}}$$

Data items above the mean have positive z-scores. Data items below the mean have negative z-scores. The z-score for the mean is 0.

Example 5 The mean weight of newborn infants is 7 pounds and the standard deviation is 0.8 pound. The weights of newborn infants are normally distributed. Find the z-score for a weight of

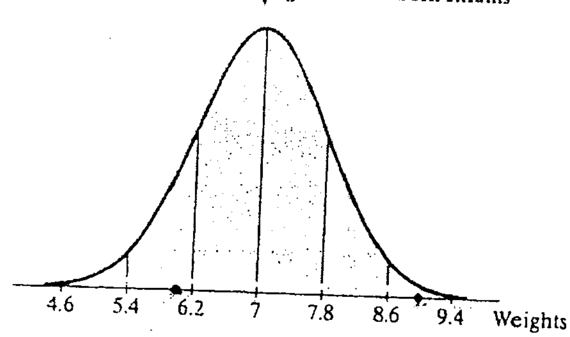
a. 9 pounds. b. 7 pounds. c. 6 pounds.

a. $\frac{9-7}{.8} = 2.5$

b. $\frac{7-7}{.8} = 0$

c. $\frac{6-7}{.8} = -1.25$

Normal Distribution of Weights of Newborn Infants



Example 6 A student scores 70 on an arithmetic test and 66 on a vocabulary test. The scores for both tests are normally distributed. The arithmetic test has a mean of 60 and a standard deviation of 20. The vocabulary test has a mean of 60 and a standard deviation of 2. On which test did the student have the better score?

arith: $z = \frac{70-60}{20} = .5$

vocab: $z = \frac{66-60}{2} = 3$

vocab test — more st. dev. above the mean

Example 7 We have seen that the SAT has a mean of 500 and a standard deviation of 100. The ACT has a mean of 18 and a standard deviation of 6. Both tests measure the same kind of ability, with scores that are normally distributed. Suppose you score 550 on the SAT and 24 on the ACT. On which test did you have the better score?

SAT: $z = \frac{550-500}{100} = .5$

ACT: $z = \frac{24-18}{6} = 1$

$\boxed{\text{ACT}}$

Example 8 Intelligence quotients (IQs) are normally distributed with a mean of 100 and a standard deviation of 15. What is the IQ corresponding to a z-score of -1.5?

$z = \frac{\text{item} - \text{mean}}{\text{SD}}$

$-1.5 = \frac{x - 100}{15}$

$-22.5 = x - 100$

$\boxed{x = 77.5}$

> PERCENTILES
>
> If n% of the items in a distribution are less than a particular data item, we say that the data item is in the nth percentile of the distribution.

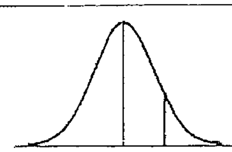
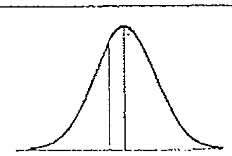
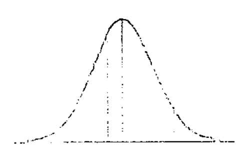
Example 9 A student scored in the 93rd percentile on the SAT. What does this mean?

the student scored higher than 93% of the test takers

z-SCORES AND PERCENTILES

z-score	Percentile	z-score	Percentile	z-score	Percentile	z-score	Percentile
−4.0	0.003	−1.0	15.87	0.0	50.00	1.1	86.43
−3.5	0.02	−0.95	17.11	0.05	51.99	1.2	88.49
−3.0	0.13	−0.90	18.41	0.10	53.98	1.3	90.32
−2.9	0.19	−0.85	19.77	0.15	55.96	1.4	91.92
−2.8	0.26	−0.80	21.19	0.20	57.93	1.5	93.32
−2.7	0.35	−0.75	22.66	0.25	59.87	1.6	94.52
−2.6	0.47	−0.70	24.20	0.30	61.79	1.7	95.54
−2.5	0.62	−0.65	25.78	0.35	63.68	1.8	96.41
−2.4	0.82	−0.60	27.43	0.40	65.54	1.9	97.13
−2.3	1.07	−0.55	29.12	0.45	67.36	2.0	97.72
−2.2	1.39	−0.50	30.85	0.50	69.15	2.1	98.21
−2.1	1.79	−0.45	32.64	0.55	70.88	2.2	98.61
−2.0	2.28	−0.40	34.46	0.60	72.57	2.3	98.93
−1.9	2.87	−0.35	36.32	0.65	74.22	2.4	99.18
−1.8	3.59	−0.30	38.21	0.70	75.80	2.5	99.38
−1.7	4.46	−0.25	40.13	0.75	77.34	2.6	99.53
−1.6	5.48	−0.20	42.07	0.80	78.81	2.7	99.65
−1.5	6.68	−0.15	44.04	0.85	80.23	2.8	99.74
−1.4	8.08	−0.10	46.02	0.90	81.59	2.9	99.81
−1.3	9.68	−0.05	48.01	0.95	82.89	3.0	99.87
−1.2	11.51	0.0	50.00	1.0	84.13	3.5	99.98
−1.1	13.57					4.0	99.997

COMPUTING PERCENTAGE OF DATA ITEMS FOR NORMAL DISTRIBUTIONS

Description of Percentage	Graph	Computation of Percentage
Percentage of data items less than a given data item with $z = b$		Use the table percentile for $z = b$ and add a % sign.
Percentage of data items greater than a given data item with $z = a$		Subtract the table percentile for $z = a$ from 100 and add a % sign.
Percentage of data items between two given data items with $z = a$ and $z = b$		Subtract the table percentile for $z = a$ from the table percentile for $z = b$ and add a % sign.

Example 10 According to the Department of Health and Education, cholesterol levels are normally distributed. For men between 18 and 24 years, the mean is 178.1 (measured in milligrams per 100 milliliters) and the standard deviation is 40.7. What percentage of men in this age range have a cholesterol level less than 239.15?

$$Z = \frac{239.15 - 178.1}{40.7} = 1.5$$

$\boxed{93.32\%}$

Example 11 Lengths of pregnancy of women are normally distributed with a mean of 266 days and a standard deviation of 16 days. What percentage of children are born from pregnancies lasting more than 274 days?

$z = \frac{274 - 266}{16} = .5$ 69.15%

100% − 69.15%

$\boxed{30.85\%}$

Example 12 According to the National Federation of State High School Associations, the amount of time that high school students work at jobs each week is normally distributed with a mean of 10.7 hours and a standard deviation of 11.2 hours. What percentage of high school students work between 5.1 and 38.7 hours each week?

$z = \frac{5.1 - 10.7}{11.2} = -.5$ 30.85%

$z = \frac{38.7 - 10.7}{11.2} = 2.5$ 99.38%

99.38% − 30.85%

$= \boxed{68.53\%}$

You try:

1. The distribution of heights of young women is approximately normal with a mean of 65 inches and a standard deviation of 2.5 inches. What percentage of young women are shorter than 68 inches?

$z = 1.2$ $\boxed{88.49\%}$

2. The Wechsler Adult Intelligence Scale (WAIS) is an IQ test. Scores on the WAIS for the 20-34 age group are normally distributed with a mean of 110 and a standard deviation of 25. What percentage of IQ scores for this age group are greater than 145?

$z = 1.4$

91.92%

100% − 91.92% $= \boxed{8.08\%}$

3. The distribution of heights of young men is approximately normal with a mean of 70 inches and a standard deviation of 2.5 inches. What percentage of young men have heights between 67 inches and 74 inches?

$z = -1.2$ $z = 1.6$

11.51% 94.52%

94.52% − 11.51%

$\boxed{83.01\%}$