Unit #3 Notes – First & Second Derivative Tests

Match each term below with its correct definition.

Important Terms:	
Critical Points	C
Absolute (Global) Maximum	I
Absolute (Global) Minimum	F
Relative (Local) Maximum	H
Relative (Local) Minimum	E
Function is Increasing	B
Function is Decreasing	J
Function is Concave Up	G
Function is Concave Down	D
Points of Inflection	A

Definitions:

A. function changes concavity/zeros of second derivative
B. derivative is positive
C. zeros of derivative or where derivative DNE
D. second derivative is negative
E. the point a graph changes from decreasing to increasing
F. minimum of entire interval
G. second derivative is positive
H. the point a graph changes from increasing to decreasing
I. maximum of entire interval
J. derivative is negative

THE FIRST DERIVATIVE TEST

rel. max|min for f

incr/decr for f

Suppose that c is a critical number of a continuous function f:

- If f' changes from positive to negative at c, then f has a local maximum at c.
- If f' changes sign from negative to positive at c, then f has local minimum at c.
- If f' does not change signs at c, then f has no local extreme value at c.
- At left endpoint a - If $f' < 0$ for $x > a$, then f has a local maximum at a. If $f' > 0$ for $x > a$, then f has a local minimum at a.
- At right endpoint b - If $f' < 0$ for $x < b$, then f has a local minimum at b. If $f' > 0$ for $x < b$, then f has a local maximum at b.

SECOND DERIVATIVE ANALYSIS

concavity & POI for f

incr/decr & max|min for f′

☐ If $f'' > 0$ then the function is concave up
☐ If $f'' < 0$ then the function is concave down

SECOND DERIVATIVE TEST

rel max|min for f

Suppose that c is a critical number of a continuous function f:

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horiz. tang. conc. down

☐ If $f'(c) = 0$ and $f''(c)<0$, then f has a local max at $x = c$

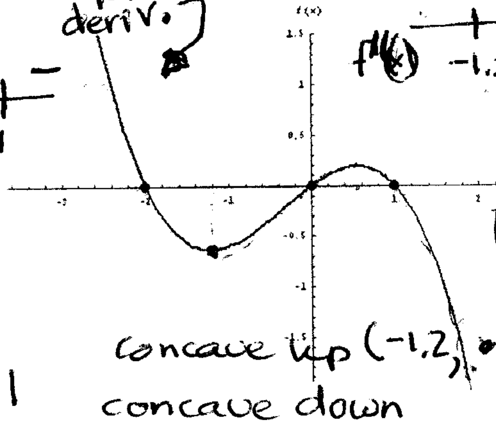
☐ If $f'(c) = 0$ and $f''(c)>0$, then f has a local min at $x = c$
☐ If $f''(c) = 0$ ~~of~~ or does not exist, the test fails. (When this happens, defer to the 1st derivative test.)

EX1) Given the graph of f ′, identify the following for f:

1. Maximums
2. Minimums
3. Intervals increasing/decreasing
4. Inflection points
5. Concavity

+ | − | + | −
f′(x) -2 0 1

1st deriv.



− | + | −
f″(x) -1.2 .5

f(x)
incr $(-\infty,-2)$ $(0,1)$
decr $(-2,0)$ $(1,\infty)$
rel. max at $x=-2$ and $x=1$
rel min at $x=0$

POI at
$x=-1.2$
and
$x=.5$

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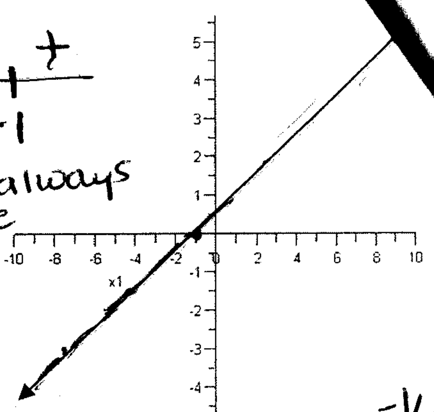
concave up $(-1.2, .5)$
concave down
$(-\infty,-1.2)$ $(.5,\infty)$

Ex2)** **Given the graph of f ', identify the following for f:

1. Maximums
2. Minimums
3. Intervals increasing/decreasing
4. Inflection points
5. Concavity



$f'(x)$ − + at -1

$f''(x)$ are always positive

$\underline{f(x)}$ decr $(-\infty, -1)$
incr $(-1, \infty)$
rel. min at $x=-1$

concave up $(-\infty,\infty)$
no POI
no rel. max

Ex3) Find all extrema on the given intervals:

a) $f(x)=x^3-6x+5$ $[-2, 3]$

$f'(x)=3x^2-6$
$3x^2-6=0$
$x^2=2$
$x=\pm\sqrt{2}$ critical values

$f'(x)$: + at $-\sqrt{2}$ − at $\sqrt{2}$ +

x	f(x)	
$-\sqrt{2}$	10.657	
$\sqrt{2}$	−.657	abs min
−2	9	
3	14	abs max

rel. max at $x=-\sqrt{2}$
rel min at $x=\sqrt{2}$

b) $f(x)=3x^{2/3}$ $[-1, 2]$

$f'(x)=3\cdot\frac{2}{3}x^{-1/3}=\frac{2}{\sqrt[3]{x}}$

undef at $x=0$

$f'(x)$: − at 0 +

rel. min at $x=0$

x	f(x)	
0	0	abs. min
−1	3	
2	$3(2)^{2/3}\approx 4.762$	abs. max

c) $f(x)=\frac{1}{\sqrt{4-x^2}}$ $(-\infty, \infty)$ → $(4-x^2)^{-1/2}$

$f'(x)=-\frac{1}{2}(4-x^2)^{-3/2}\cdot -2x$
$=\frac{x}{(4-x^2)^{3/2}}$

$f'(x)=0$ when $x=0$
$f'(x)$ undef when $x=\pm 2$

$f'(x)$: not dif at -2 − at 0 + at 2 not dif.

rel. min at $x=0$

abs. min at $(0,\frac{1}{2})$
no abs. max

Ex4) Find all the intervals where $f(x)=4x^3-3x^2-18x+6$ is increasing & all intervals where f is decreasing.

$f'(x)=12x^2-6x-18$
$12x^2-6x-18=0$
$6(2x^2-x-3)=0$
$6(2x-3)(x+1)=0$

$x=3/2$ $x=-1$

$f'(x)$: + at -1 − at $3/2$ +

f incr $(-\infty,-1)$
f incr $(3/2,\infty)$
f decr $(-1, 3/2)$

Ex5) ~~Find the~~ relative extrema of $y=\sin(x)-2\cos(x)$ in the interval $[0, 2\pi]$. Justify your answer.

$y'=\cos x+2\sin x$
$\cos x+2\sin x=0$ when
$x=2.678$ and 5.820
(use a calculator)

$0\,[$ + at 2.678 − at 5.820 + $]\,2\pi$
$f'(x)$

f rel. max at $x=2.678$
f rel min at $x=5.820$

Ex6) Find the points of inflection for $g(x)=3x^4-8x^3+6x^2$. Justify your answer.

$g'(x)=12x^3-24x^2+12x$
$g''(x)=36x^2-48x+12$
$36x^2-48x+12=0$
$12(3x^2-4x+1)=0$
$12(3x-1)(x-1)=0$
$x=1/3$ $x=1$

18 $f''(x)$: + at $1/3$ − at 1 +

POI
$(1/3, 11/27)$
$(1,1)$

f(x) has POI at $x=1/3$ & $x=1$