You should be able to . . .

- Use an initial condition to find a particular solution to a differential equation
- Sketch a slope field and use it to approximate a solution of a differential equation
- Solve a problem involving a logistic equation
- Use Euler's Method to approximate a solution to a differential equation

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Differential Equations

A **solution** has the form $y = f(x)$.

general solution- has C in it.

particular solution- find C using the **initial conditions**.

Ex 1) Find the general solution: $\frac{dy}{dx} = \frac{y}{x-1}$

$(x-1)dy = y\,dx$

$\int \frac{dy}{y} = \int \frac{dx}{x-1}$

$e^{\ln|y|} = e^{\ln|x-1| + C} = e^{\ln|x-1|} \cdot e^{C}$

constant

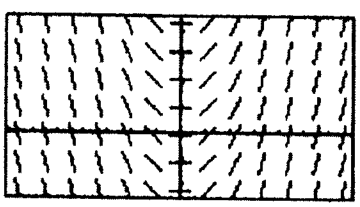
$y = C|x-1|$

Slope Fields

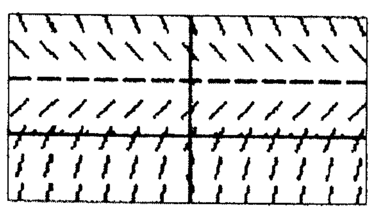
- A **slope field** is a visual representation of the solution of a differential equation.
- A **slope field** shows the general "flow" of a differential equation's solution.
- Often, **slope fields** are used in lieu of actually solving the differential equation.

Ex 2) Match each slope field to its differential equation.

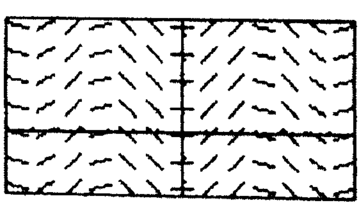
A)



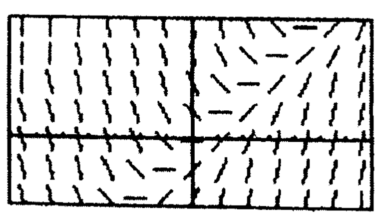
B)



C)



D)



$\frac{dy}{dx} = \sin x$ C

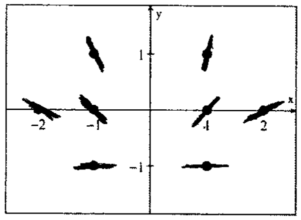
$\frac{dy}{dx} = x - y$ D

$\frac{dy}{dx} = 2 - y$ B

$\frac{dy}{dx} = x$ A

Ex 3) Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$ where $x \neq 0$.

a) Sketch a slope field for the given differential equation at the 8 points indicated.



b) Find the particular solution $y = f(x)$ to the differential equation if $f(-1) = 1$.

$x\,dy = (1+y)\,dx$

$\int \frac{dy}{1+y} = \int \frac{dx}{x}$

$\ln|1+y| = \ln|x| + C$

$\ln|1+1| = \ln|-1| + C$

$\ln 2 = C$

$\ln|1+y| = \ln|x| + \ln 2$

$e^{\ln|1+y|} = e^{\ln 2|x|}$

$1+y = 2|x|$

$y = -1 + 2|x|$

Logistic Growth

Differential Equation:

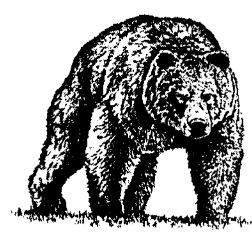
$$\frac{dy}{dx} = ky\left(1 - \frac{y}{L}\right)$$

where k = Growth rate & L = Carrying Capacity

Solution to a logistic differential equation:

$$P(t) = \frac{L}{1 + Ae^{-kt}} \quad \text{where} \quad A = \frac{L - P(0)}{P(0)}$$

Ex 4) Ten grizzly bears were introduced to a national park 10 years ago. There are 23 bears in the park at the present time. The park can support a maximum of 100 bears. Assuming logistic growth model, when will the bear population reach 50? 75?



$(t, p(t))$

$(0, 10)$

$(10, 23)$

$L = 100$

$A = \frac{100 - 10}{10} = 9$

$p(t) = \frac{100}{1 + 9e^{-kt}}$

$23 = \frac{100}{1 + 9e^{-k(10)}}$

$k = .09889134$

$50 = \frac{100}{1 + 9e^{-.099t}}$

in 22.219 yrs

$75 = \frac{100}{1 + 9e^{-.099t}}$

33.328 yrs

Euler's Method

Euler's Method is an iterative process which gives us numerical method to approximate the particular solution to a differential equation.

$$y_1 = y_0 + f(x_0, y_0)\Delta x$$

$y - y_1 = m(x - x_1)$

$\underset{\text{new } y}{y} = \underset{\text{old } y}{y_1} + \underset{dy/dx}{m}\underbrace{(x - x_1)}_{\Delta x \text{ step size}}$

Ex 5) If $\frac{dy}{dx} = \frac{x-y}{2y}$ and $y = -2$ when $x = 3$, approximate the value of y when $x = 3.2$ using Euler's method with a step size of 0.1.

x	y	$dy/dx = \frac{x-y}{2y}$
3	-2	$\frac{3-(-2)}{2(-2)} = -\frac{5}{4}$
3.1	-2.125	$-209/170$
3.2	$\boxed{\frac{-7643}{3400}}$	

$-2 + \frac{-5}{4}(.1) = -2.125$

$-2.125 + \frac{-209}{170}(.1) = \boxed{\frac{-7643}{3400} \text{ OR } -2.248}$