

Integration By Tables

➤ idea is to recognize which rules and/or technique to apply

Example 1 Evaluate: $\int \frac{dx}{x\sqrt{x-1}} \rightarrow \int \frac{du}{u\sqrt{u+a}}$

#17 $u=x$ $a=-1$ $b=1$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\frac{2}{\sqrt{1}} \arctan \sqrt{\frac{-1+x}{1}} + C = 2 \arctan \sqrt{-1+x} + C$$

Example 2 Evaluate: $\int x\sqrt{x^4-9} dx \rightarrow \frac{1}{2} \int \sqrt{u^2-a^2} du$

#26

$$u^2 = x^4 \quad a^2 = 9$$

$$u = x^2 \quad a = 3$$

$$\frac{du}{dx} = 2x$$

$$\frac{1}{2} du = x dx$$

$$\frac{1}{2} \left[\frac{1}{2} \left[x^2 \sqrt{x^4-9} - 9 \ln |x^2 + \sqrt{x^4-9}| \right] \right] + C$$

Example 3 Evaluate: $\int \frac{x}{1+e^{-x^2}} dx \rightarrow -\frac{1}{2} \int \frac{du}{1+e^u}$

#84

$$u = -x^2$$

$$\frac{du}{dx} = -2x$$

$$-\frac{1}{2} du = x dx$$

$$-\frac{1}{2} \left[-x^2 - \ln(1+e^{-x^2}) \right] + C$$

Example 4 Evaluate: $\int x^3 \sin x dx \rightarrow \int u^n \sin u du$

54

$$u = x \quad n = 3 \\ du = dx$$

$$-x^3 \cos x + 3 \left[x^2 \sin x - 2 (\sin x - x \cos x) \right] + C$$

$$-x^3 \cos x + 3 \int x^2 \cos x dx$$

55 $n=2$

$$x^2 \sin x - 2 \int x' \sin x dx$$

52

Example 5 Evaluate: $\int \frac{\sqrt{3-5x}}{2x} dx \rightarrow \frac{1}{2} \int \frac{\sqrt{a+bu}}{u} du \sin x - x \cos x + C$

19

$$a = 3 \quad b = -5 \quad u = x \\ du = dx$$

$$\frac{1}{2} \left[2 \sqrt{3-5x} + 3 \int \frac{1}{x \sqrt{3-5x}} dx \right]$$

17

$$\frac{1}{\sqrt{3}} \ln \left| \frac{\sqrt{3-5x} - \sqrt{3}}{\sqrt{3-5x} + \sqrt{3}} \right| + C$$

$$\sqrt{3-5x} + \frac{3}{2\sqrt{3}} \ln \left| \frac{\sqrt{3-5x} - \sqrt{3}}{\sqrt{3-5x} + \sqrt{3}} \right| + C$$

Example 6 Evaluate: $\int \frac{\sin 2x}{2 + \cos x} dx = \int \frac{2 \sin x \cos x}{2 + \cos x} dx$

3

$$a = 2 \quad b = 1 \quad u = \cos x$$

$$du = -\sin x dx$$

$$-du = \sin x dx$$

$$-2 \int \frac{u du}{a + bu}$$

$$-2 \left[\frac{1}{12} (\cos x - 2 \ln |2 + \cos x|) \right] + C$$

$$-2 \cos x + 4 \ln |2 + \cos x| + C$$