Natural Log – Integration

rule: $\int \frac{1}{x}\,dx = \ln|x| + C$

EX1 Find:

A. $\int \frac{2}{x-5}\,dx = 2\int \frac{du}{u} = 2\ln|u|+c$

$= 2\ln|x-5|+C$

$u = x-5$

$\frac{du}{dx} = 1 \quad du = dx$

B. $\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u|+C$

$\frac{1}{2}\ln|x^2+1|+c$

or

$\frac{1}{2}\ln(x^2+1)+c$

$u = x^2+1$

$\frac{du}{dx} = 2x \quad du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

C. $\int \frac{x^2-4}{x}\,dx = \int \left(x - \frac{4}{x}\right)dx = \frac{1}{2}x^2 - 4\ln|x| + C$

D. $\int \frac{\sec x \tan x}{\sec x - 1}\,dx = \int \frac{du}{u} = \ln|u|+c$

$\ln|\sec x - 1| + c$

$u = \sec x - 1$

$\frac{du}{dx} = \sec x \tan x$

$du = \sec x \tan x\,dx$

EX 2 Evaluate

A. $\int_0^2 \frac{2}{5x+1}\,dx$ $\qquad \frac{1}{5}\cdot 2\int \frac{du}{u} = \frac{2}{5}\ln|u| + C$

$$\frac{2}{5}\ln|5x+1| + C\,\Big|_0^2$$

$u = 5x+1$

$\frac{du}{dx} = 5 \quad \frac{1}{5}du = dx$

$$\frac{2}{5}\ln|11| + c - \Big(\frac{2}{5}\underbrace{\ln|1|}_{0} + c\Big)$$

$$\boxed{\frac{2}{5}\ln 11}$$

B. $\int_e^{e^2} \frac{1}{x\ln x}\,dx$ $\qquad \int \frac{du}{u} = \ln|u| + C$

$$\ln|\ln x| + C\,\Big|_e^{e^2}$$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x} \quad du = \frac{1}{x}dx$

$$\ln|\ln e^2| + c - (\ln|\ln e| + c)$$

$\ln e^2 \downarrow 2\underbrace{\ln e}_{1}$ $\qquad \ln e \downarrow 1$

$$\boxed{\ln 2}$$

EX 3 Solve the differential eqn if r contains $(\pi, 4)$

$$\frac{dr}{dt} = \frac{\sec^2 t}{\tan t + 1}$$

$$\int 1\,dr = \int \frac{\sec^2 t}{\tan t + 1}\,dt$$

$u = \tan t + 1$

$\frac{du}{dt} = \sec^2 t$

$\int \frac{du}{u} = \ln|u| + C$

$r = \ln|\tan t + 1| + C$

$4 = \ln|\tan \pi + 1| + C$

$4 = \ln 1 + C$

$4 = C$

$r = \ln|\tan t + 1| + 4$

<u>EX 4</u> $f(x) = \tan x$. Find the average value over $[0, \pi/4]$.

$$\frac{1}{\pi/4 - 0} \int_0^{\pi/4} \tan x \, dx = \frac{4}{\pi}\left[-\ln|\cos x| + C \Big|_0^{\pi/4} \right]$$

$$= \frac{4}{\pi}\left[-\ln|\cos \tfrac{\pi}{4}| + C - (-\ln|\cos 0| + C) \right]$$

$$= \frac{4}{\pi}\left[-\ln \frac{\sqrt{2}}{2} + \underbrace{\ln 1}_{0} \right]$$

$$= \frac{-4 \ln \frac{\sqrt{2}}{2}}{\pi}$$